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Multiple Attribute Decision Making Based on New Transformation Method of Connection Numbers, Set Pair Analysis Theory, and New Score Function of Interval-Valued Intuitionistic Fuzzy Values

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In this paper, we propose a new multiple attribute decision-making (MADM) methodology using our proposed new connection numbers (CNs) transformation method for interval-valued intuitionistic fuzzy (IVIF) values, our developed score function for IVIF values, the set pair analysis theory, the IVIF positive ideal solution, and the normalized exponential Hamming distance between CNs, where the weights of attributes are represented by IVIF values. Our proposed CNs transformation methodology for IVIF values can overcome the drawback of the existing CNs transformation methodology for IVIF values. Moreover, our proposed MADM methodology can overcome the shortcomings of the existing MADM methodologies in IVIF environments. Our proposed MADM methodology offers a very useful approach for dealing with MADM problems in IVIF environments.

Keywords: connection numbers; decision matrix; IVIF values; MADM; score function

1. INTRODUCTION

In [2], Atanassov proposed the concept of intuitionistic fuzzy sets (IFSs) by extending the concept of fuzzy sets [15]. In [1], Atanassov and Gargov presented the theory of interval-valued intuitionistic fuzzy sets (IVIFSs) by expanding the concepts of IFSs. In recent years, some interval-valued intuitionistic fuzzy (IVIF) multiple attribute decision making (MADM) methodologies [3], [5], [6], [9], [12], [13] have been presented based on IVIFSs. However, in this paper, we find that

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(1) The connection numbers (CNs) transformation methodology of IVIF values presented in [9] has the shortcoming that it transforms two different IVIF values into the same CN in some situations.
(2) The MADM methodologies presented in [3] and [9] in IVIF environments have the drawbacks that they cannot distinguish the preference orders (PFOs) of alternatives in some circumstances. Moreover, the MADM methodologies presented in [12] and [13] in IVIF environments have the drawback that they cannot get the PFOs of alternatives in some circumstances.

Therefore, it is necessary to develop a new CNs transformation methodology for IVIF values to overcome the drawbacks of the CNs transformation methodology for IVIF values presented in [9]. Moreover, it is also necessary to develop a new MADM methodology to overcome the drawbacks of the MADM methodologies presented in [3], [9], [12] and [13] in IVIF environments.

In this paper, we propose a new MADM methodology based on our proposed new CNs transformation methodology of IVIF values, our developed score function (SCFN) [4] of IVIF values, the IVIF positive ideal solution (PIS) [11], the set pair analysis (SPA) theory [16], and the normalized exponential Hamming distance [8] between CNs, where the weights of attributes are represented by IVIF values. Our proposed CNs transformation methodology for IVIF values is able to conquer the drawback of the CNs transformation methodology of IVIF values presented in [9]. Our proposed MADM methodology is able to conquer the drawbacks of the MADM methodologies presented in [3], [9], [12] and [13] in IVIF environments.

2. PRELIMINARIES

Definition 2.1 **[1].** Let $Q=\{< x_t, \mu_Q(x_t), \nu_Q(x_t) > | x_t \in X\}$ be an IVIFS and let $X=\{x_1, x_2, \cdots, x_v\}$ be the universe of discourse, where $\mu_Q(x_t)$ and $\nu_Q(x_t)$ indicate the grades of the membership and the non-membership of x_t in the IVIFS Q, respectively, $\mu_Q(x_t)=[d_t^L, d_t^U]$, $\nu_Q(x_t)=[g_t^L, g_t^U]$, $0 \le d_t^L \le d_t^U \le 1$, $0 \le g_t^L \le g_t^U \le 1$, $0 \le d_t^U + g_t^U \le 1$, and $1 \le t \le v$.

Definition 2.2 **[14].** Let $Q=\{< x_t, \mu_Q(x_t), \nu_Q(x_t) > | x_t \in X\}$ be an IVIFS in the universe of discourse $X=\{x_1, x_2, \cdots, x_v\}$, where $\mu_Q(x_t)=[d_t^L, d_t^U]$, $\nu_Q(x_t)=[g_t^L, g_t^U]$ and $1 \le t \le v$. Then, $([d_t^L, d_t^U], [g_t^L, g_t^U])$ is called an IVIF value, where $1 \le t \le v$.

Definition 2.3 **[1], [9].** Let $\tilde{\rho}_1=([d_1^L, d_1^U], [g_1^L, g_1^U])$ and $\tilde{\rho}_2=([d_2^L, d_2^U], [g_2^L, g_2^U])$ be two IVIF values. Then,

(i) If $d_1^L \le d_2^L$, $d_1^U \le d_2^U$, $g_1^L \le g_2^L$ and $g_1^U \le g_2^U$, then $\tilde{\rho}_1 \le \tilde{\rho}_2$.

(ii) If $d_1^L = d_2^L$, $d_1^U = d_2^U$, $g_1^L = g_2^L$ and $g_1^U = g_2^U$, then $\tilde{\rho}_1 = \tilde{\rho}_2$.

Definition 2.4 **[7], [8], [9], [16].** Let $S(\theta_1, \theta_2)$ be a set pair between two interrelated sets θ_1 and θ_2 for the problem P having T features, where M denotes the number of mu-

tual features between the sets θ_1 and θ_2; O denotes the number of opposite features between the sets θ_1 and θ_2; $D = T - M - O$ denotes the number of features which are neither mutual features nor opposite features between the sets θ_1 and θ_2. The CN for the set pair $S(\theta_1, \theta_2)$ of two interrelated sets θ_1 and θ_2 for the problem P is represented by $k(S, P)$, where

$$k(S,P)=\left(\frac{M}{T}\right)+\left(\frac{D}{T}\right)i+\left(\frac{O}{T}\right)j, \tag{1}$$

the ratio "$\frac{M}{T}$" indicates the degree of identity between the sets θ_1 and θ_2, the ratio "$\frac{D}{T}$" indicates the degree of discrepancy between the sets θ_1 and θ_2, the ratio "$\frac{O}{T}$" indicates the degree of contrary between the sets θ_1 and θ_2, i is the coefficient of the degree of discrepancy, $i \in [-1, 1]$, j is the coefficient of the degree of contrary, and $j = -1$.

For simplicity, let $\alpha=\frac{M}{T}$, $\beta=\frac{D}{T}$ and $\gamma=\frac{O}{T}$ denote the degree of identity, the degree of discrepancy and the degree of contrary between the sets θ_1 and θ_2, respectively. Then, Eq. (1) can be rewritten as follows:

$$k=\alpha+\beta i+\gamma j, \tag{2}$$

where $\alpha \in [0, 1]$, $\beta \in [0, 1]$, $\gamma \in [0, 1]$, and $\alpha+\beta+\gamma=1$.

Definition 2.5 **[9].** The corresponding CN k_A of the IVIF value $\tilde{\rho}=\left(\left[d_A^L, d_A^U\right],\left[g_A^L, g_A^U\right]\right)$ obtained by Kumar and Chen's approach is defined below:

$$k_A=\alpha_A+\beta_A i+\gamma_A j,$$

where

$$\alpha_A=\frac{d_A^L\left(2-d_A^U-g_A^U\right)+d_A^U\left(2-d_A^L-g_A^L\right)}{2}, \tag{3}$$

$$\beta_A=\frac{g_A^L\left(2-d_A^U-g_A^U\right)+g_A^U\left(2-d_A^L-g_A^L\right)}{2}, \tag{4}$$

$$\begin{aligned}\gamma_A &= 1-\alpha_A-\beta_A \\ &= 1-\frac{d_A^L\left(2-d_A^U-g_A^U\right)+d_A^U\left(2-d_A^L-g_A^L\right)}{2}-\frac{g_A^L\left(2-d_A^U-g_A^U\right)+g_A^U\left(2-d_A^L-g_A^L\right)}{2}\end{aligned} \tag{5}$$

$\alpha_A \in [0,1]$, $\beta_A \in [0,1]$, $\gamma_A \in [0,1]$, $\alpha_A+\beta_A+\gamma_A=1$, $i \in [-1, 1]$, and $j = -1$.

Definition 2.6 **[8], [10].** Let $k_1=\alpha_1+\beta_1 i+\gamma_1 j$ and $k_2=\alpha_2+\beta_2 i+\gamma_2 j$ be two CNs. Then,

(i) $k_1=k_2$ if and only if $\alpha_1=\alpha_2$, $\beta_1=\beta_2$ and $\gamma_1=\gamma_2$.

(ii) $k_1 \le k_2$ if $\alpha_1 \le \alpha_2$ and $\beta_1 \ge \beta_2$.

Definition 2.7 **[8].** The normalized exponential Hamming distance $D\left(k_A, k_B\right)$ between the CNs $k_A=\alpha_A+\beta_A i+\gamma_A j$ and $k_B=\alpha_B+\beta_B i+\gamma_B j$ is defined as follows:

$$D\left(k_A,k_B\right)=\frac{1-\exp\left[-\frac{1}{3}\left(\left|\sqrt{\alpha_A}-\sqrt{\alpha_B}\right|+\left|\sqrt{\beta_A}-\sqrt{\beta_B}\right|+\left|\sqrt{\gamma_A}-\sqrt{\gamma_B}\right|\right)\right]}{1-\exp(-1)}, \tag{6}$$

where $D(k_A, k_B) \in [0,1]$.

***Definition 2.8* [11].** Let $\omega_1, \omega_2, \cdots$, and ω_r be alternatives and let $\varphi_1, \varphi_2, \cdots$, and φ_r be attributes. Let P_1 be a set of benefit-type attributes and let P_2 be a set of cost-type attributes, where $P_1 \cap P_2 = \emptyset$, and $P_1 \cup P_2 = P$. Let $\tilde{\rho}_{qp} = \left(\left[d_{qp}^L, d_{qp}^U \right], \left[g_{qp}^L, g_{qp}^U \right] \right)$ be an evaluating IVIF value of alternative ω_q with respect to attribute φ_p, where $1 \le q \le r$ and $1 \le p \le s$. Then, the IVIF positive ideal solution (PIS) $\tilde{A}^+$ is defined as follows:

$$\tilde{A}^+ = \left\{ \tilde{\rho}_1^+, \tilde{\rho}_2^+, \cdots, \tilde{\rho}_n^+ \right\} = \left\{ \left(\max_q \tilde{\rho}_{qp} \middle| \varphi_p \in P_1 \right), \left(\min_q \tilde{\rho}_{qp} \middle| \varphi_p \in P_2 \right) \right\}, \tag{7}$$

where if $\varphi_p \in P_1$, then let $\tilde{\rho}_p^+ = \left(\left[\max_q d_{qp}^L, \max_q d_{qp}^U \right], \left[\min_q g_{qp}^L, \min_q g_{qp}^U \right] \right) = \left(\left[d_p^{L^+}, d_p^{U^+} \right], \left[g_p^{L^+}, g_p^{U^+} \right] \right)$; if $\varphi_p \in P_2$, then let $\tilde{\rho}_p^+ = \left(\left[\min_q d_{qp}^L, \min_q d_{qp}^U \right], \left[\max_q g_{qp}^L, \max_q g_{qp}^U \right] \right) = \left(\left[d_p^{L^+}, d_p^{U^+} \right], \left[g_p^{L^+}, g_p^{U^+} \right] \right)$, where $1 \le q \le r$ and $1 \le p \le s$.

3. OUR PROPOSED CONNECTION NUMBERS TRANSFORMATION METHOD FOR INTERVAL-VALUED INTUITIONISTIC FUZZY VALUES

In this section, we propose a new method for transforming an IVIF value into a connection number (CN) to overcome the shortcoming of Kumar and Chen's approach [9] for transforming an IVIF value into a CN, which has the drawback of transforming two different IVIF values into the same CN.

***Definition 3.1* [6].** The corresponding CN k_A of the IVIF value $\tilde{\rho}_A = \left(\left[d_A^L, d_A^U \right], \left[g_A^L, g_A^U \right] \right)$ obtained by our proposed approach is defined as follows:

$$k_A = \alpha_A + \beta_A i + \gamma_A j,$$

where $i \in [-1, 1]$, $j = -1$,

$$\alpha_A = \frac{d_A^L \left(1 + d_A^U - g_A^U\right) + d_A^U \left(1 + d_A^L - g_A^L\right)}{4}, \tag{8}$$

$$\beta_A = \frac{4 - \left(d_A^L + d_A^U + g_A^L + g_A^U + 2d_A^L d_A^U + 2g_A^L g_A^U - 2d_A^L g_A^U - 2d_A^U g_A^L\right)}{4}, \tag{9}$$

$$\gamma_A = \frac{g_A^L \left(1 - d_A^U + g_A^U\right) + g_A^U \left(1 - d_A^L + g_A^L\right)}{4}, \tag{10}$$

$\alpha_A \in [0,1]$, $\beta_A \in [0,1]$, $\gamma_A \in [0,1]$, and $\alpha_A + \beta_A + \gamma_A = 1$.

Example 3.1. Let $\tilde{\rho}_A = \left([0.2,1], [0,0] \right)$ and $\tilde{\rho}_B = \left([0.9,1], [0,0] \right)$ be two different IVIF values. Based on Eqs. (3), (4) and (5), the corresponding CNs k_A and k_B of the IVIF val-

ues $\tilde{\rho}_A = ([0.2,1],[0,0])$ and $\tilde{\rho}_B = ([0.9,1],[0,0])$, respectively, obtained by Kumar and Chen's methodology [9] are $k_A = 1 + 0i + 0j$ and $k_B = 1 + 0i + 0j$, where $k_A = k_B = 1 + 0i + 0j$. Therefore, Kumar and Chen's methodology [9] has the drawback that it transforms two different IVIF values into the same CN, which is not reasonable. On the other hand, based on Eqs. (8), (9) and (10), the corresponding CNs k_A and k_B of the IVIF values $\tilde{\rho}_A = ([0.2,1],[0,0])$ and $\tilde{\rho}_B = ([0.9,1],[0,0])$ obtained by our proposed methodology are $k_A = 0.400 + 0.600i + 0j$ and $k_B = 0.925 + 0.075i + 0j$, respectively, where k_A and k_B are two different CNs. Therefore, our proposed methodology for transforming an IVIF value into a CN is able to overcome the drawback of Kumar and Chen's methodology [9] for transforming an IVIF value into a CN in this situation, where Kumar and Chen's methodology [9] has the shortcoming of transforming two different IVIF values into the same CN.

4. OUR PROPOSED MADM METHODOLOGY

In [4], we have developed a score function (SCFN) *BHF* for IVIF values, shown as follows.

Definition 4.1 **[4].** Our proposed SCFN *BHF* of the IVIF value $\tilde{\rho} = ([d^L, d^U], [g^L, g^U])$ is defined as below:

$$BHF(\tilde{\rho}) = \frac{\tanh(d^L) + \tanh(d^U) + \tanh(1-g^L) + \tanh(1-g^U)}{2} + \frac{\ln(d^L + (1-g^L) + 1) + \ln(d^U + (1-g^U) + 1)}{2} \tag{11}$$

where $BHF(\tilde{\rho}) \in [0, 2\tanh(1) + \ln(3)]$. The larger the value of $BHF(\tilde{\rho})$, the larger the IVIF value $\tilde{\rho}$.

Let us consider *s* attributes $\varphi_1, \varphi_2, \cdots$, and φ_s and let us consider *r* alternatives $\omega_1, \omega_2, \cdots$, and ω_r. Let $\tilde{A} = (\tilde{\pi}_{qp})_{r\times s}$ be the decision matrix (DCMX) offered by the decision maker (DCMK), where $\tilde{\pi}_{qp} = ([\tilde{d}_{qp}^L, \tilde{d}_{qp}^U], [\tilde{g}_{qp}^L, \tilde{g}_{qp}^U])$ is an evaluating IVIF value of alternative ω_q with respect to attribute φ_p given by the DCMK, $1 \le q \le r$ and $1 \le p \le s$. Let $\tilde{f}_p$ be the IVIF weight of attribute φ_p provided by the DCMK, where $\tilde{f}_p = ([a_p^L, a_p^U], [q_p^L, q_p^U])$ and $1 \le p \le s$. Let $\tilde{A} = (\tilde{\pi}_{qp})_{r\times s}$ be the DCMX given by the DCMK. Our proposed MADM methodology is presented as follows:

Step 1: According to [9], convert the DCMX $\tilde{A} = (\tilde{\pi}_{qp})_{r\times s} = \left(([\tilde{d}_{qp}^L, \tilde{d}_{qp}^U], [\tilde{g}_{qp}^L, \tilde{g}_{qp}^U])\right)_{r\times s}$ given by the DCMK into the normalized

DCMX $A=\left(h_{qp}\right)_{r\times s}=\left(\left(\left[d_{qp}^{L},d_{qp}^{U}\right],\left[g_{qp}^{L},g_{qp}^{U}\right]\right)\right)_{r\times s}$, where $1 \le q \le r$ and $1 \le p \le s$, shown as follows:

$$\tilde{h}_{qp}=\begin{cases}\left(\left[\tilde{d}_{qp}^{L},\tilde{d}_{qp}^{U}\right],\left[\tilde{g}_{qp}^{L},\tilde{g}_{qp}^{U}\right]\right), \text{if } \varphi_p \text{ is a benefit-type attribute}\\ \left(\left[\tilde{g}_{qp}^{L},\tilde{g}_{qp}^{U}\right],\left[\tilde{d}_{qp}^{L},\tilde{d}_{qp}^{U}\right]\right), \text{if } \varphi_p \text{ is a cost-type attribute}\end{cases} \tag{12}$$

where $\tilde{h}_{qp}$ is an IVIF value, $1 \le q \le r$ and $1 \le p \le s$.

Step 2: Based on our proposed CNs transformation methodology of IVIF values shown in Eqs. (8), (9) and (10), convert each IVIF value $\tilde{h}_{qp}$ appearing in the obtained normalized DCMX $A=\left(h_{qp}\right)_{r\times s}=\left(\left(\left[d_{qp}^{L},d_{qp}^{U}\right],\left[g_{qp}^{L},g_{qp}^{U}\right]\right)\right)_{r\times s}$ into a CN $k_{qp}=\alpha_{qp}+\beta_{qp}i+\gamma_{qp}j$ to construct the CN DCMX $N=\left(k_{qp}\right)_{r\times s}=\left(\alpha_{qp}+\beta_{qp}i+\gamma_{qp}j\right)_{r\times s}$, where $i \in [-1, 1]$, $j = -1$,

$$\alpha_{qp}=\frac{d_{qp}^{L}\left(1+d_{qp}^{U}-g_{qp}^{U}\right)+d_{qp}^{U}\left(1+d_{qp}^{L}-g_{qp}^{L}\right)}{4}, \tag{13}$$

$$\beta_{qp}=\frac{4-\left(d_{qp}^{L}+d_{qp}^{U}+g_{qp}^{L}+g_{qp}^{U}+2d_{qp}^{L}d_{qp}^{U}+2g_{qp}^{L}g_{qp}^{U}-2d_{qp}^{L}g_{qp}^{U}-2d_{qp}^{U}g_{qp}^{L}\right)}{4}, \tag{14}$$

$$\gamma_{qp}=\frac{g_{qp}^{L}\left(1-d_{qp}^{U}+g_{qp}^{U}\right)+g_{qp}^{U}\left(1-d_{qp}^{L}+g_{qp}^{L}\right)}{4}, \tag{15}$$

$\beta_{qp}=1-\alpha_{qp}-\gamma_{qp}$, $\alpha_{qp}\in[0,1]$, $\beta_{qp}\in[0,1]$, $\gamma_{qp}\in[0,1]$, $1 \le q \le r$, and $1 \le p \le s$.

Step 3: Based on our proposed SCFN *BHF* of IVIF values shown in Eq. (11), convert the IVIF weight $\tilde{f}_p=\left(\left[a_p^{L},a_p^{U}\right],\left[q_p^{L},q_p^{U}\right]\right)$ of attribute φ_p given by the DCMK into a crisp weight cw_p, where $1 \le p \le s$, shown as follows:

$$cw_p=BHF\left(\tilde{\alpha}_p\right)=\frac{\tanh\left(a_p^{L}\right)+\tanh\left(a_p^{U}\right)+\tanh\left(1-q_p^{L}\right)+\tanh\left(1-q_p^{U}\right)}{2}+\frac{\ln\left(a_p^{L}+\left(1-q_p^{L}\right)+1\right)+\ln\left(a_p^{U}+\left(1-q_p^{U}\right)+1\right)}{2} \tag{16}$$

where $BHF\left(\tilde{\alpha}_p\right)\in\left[0,2\tanh(1)+\ln(3)\right]$ and $1 \le p \le s$.

Step 4: Convert the crisp weight cw_p of attribute φ_p into the normalized crisp weight $\overline{cw}$, shown below:

$$\overline{cw}_p=\frac{cw_p}{\sum_{p=1}^{s}cw_p}, \tag{17}$$

where $\overline{cw}_p \in [0, 1]$ and $1 \le p \le s$.

Step 5: According to Eq. (7), get the IVIF PIS $A^+ = \left(h_p{}^+\right)_{1\times s}$ from the normalized DCMX $A = \left(h_{qp}\right)_{r\times s}$, where $h_{qp} = \left(\left[d_{qp}^L, d_{qp}^U\right], \left[g_{qp}^L, g_{qp}^U\right]\right)$ and $h_p{}^+ = \left(\left[d_p^{L^+}, d_p^{U^+}\right], \left[g_p^{L^+}, g_p^{U^+}\right]\right)$ are IVIF values, $d_p^{L^+} = \max\limits_q \left\{d_{qp}^L\right\}$, $d_p^{U^+} = \max\limits_q \left\{d_{qp}^U\right\}$, $g_p^{L^+} = \min\limits_q \left\{g_{qp}^L\right\}$, $g_p^{U^+} = \min\limits_q \left\{g_{qp}^U\right\}$, $1 \le q \le r$, and $1 \le p \le s$.

Step 6: According to our proposed CNs transformation methodology of IVIFVs shown in Eqs. (8), (9) and (10), transform each IVIF value $h_p{}^+ = \left(\left[d_p^{L^+}, d_p^{U^+}\right], \left[g_p^{L^+}, g_p^{U^+}\right]\right)$ appearing in the obtained IVIF PIS $A^+ = \left(h_p{}^+\right)_{1\times s}$ into a CN $f\max_p = \alpha_p{}^+ + \beta_p{}^+ i + \gamma_p{}^+ j$ to construct the CN IVIF PIS $N\max = \left(f\max_p\right)_{1\times s} = \left(\alpha_p{}^+ + \beta_p{}^+ i + \gamma_p{}^+ j\right)_{1\times s}$, where

$$\alpha_p{}^+ = \frac{d_{qp}^{L^+}\left(1 + d_{qp}^{U^+} - g_{qp}^{U^+}\right) + d_{qp}^{U^+}\left(1 + d_{qp}^{L^+} - g_{qp}^{L^+}\right)}{4}, \tag{18}$$

$$\beta_p{}^+ = \frac{4 - \left(d_p^{L^+} + d_p^{U^+} + g_p^{L^+} + g_p^{U^+} + 2d_p^{L^+}d_p^{U^+} + 2g_p^{L^+}g_p^{U^+} - 2d_p^{L^+}g_p^{U^+} - 2d_p^{U^+}g_p^{L^+}\right)}{4}, \tag{19}$$

$$\gamma_p{}^+ = \frac{g_p^{L^+}\left(1 - d_p^{U^+} + g_p^{U^+}\right) + g_p^{U^+}\left(1 - d_p^{L^+} + g_p^{L^+}\right)}{4}, \tag{20}$$

$\beta_p{}^+ = 1 - \alpha_p{}^+ - \gamma_p{}^+$, $\alpha_p{}^+ \in [0,1]$, $\beta_p{}^+ \in [0,1]$, $\gamma_p{}^+ \in [0,1]$, $i \in$ [-1, 1], j = -1, and $1 \le p \le s$.

Step 7: Based on Eq. (6), calculate the normalized exponential Hamming distance $d\max_{qp}$ [8] between the CN $k_{qp} = \alpha_{qp} + \beta_{qp} i + \gamma_{qp} j$ appeared in the CN DCMX $N = \left(k_{qp}\right)_{r\times s} = \left(\alpha_{qp} + \beta_{qp} i + \gamma_{qp} j\right)_{r\times s}$ obtained in **Step 2** and the CN $f\max_p = \alpha_p{}^+ + \beta_p{}^+ i + \gamma_p{}^+ j$ appeared in the CN IVIF PIS $N\max = \left(f\max_p\right)_{1\times s} = \left(\alpha_p{}^+ + \beta_p{}^+ i + \gamma_p{}^+ j\right)_{1\times s}$ obtained in **Step 6**, where $1 \le q \le r$ and $1 \le p \le s$, shown as follows:

$$d\max_{qp} = \frac{1 - \exp\left[-\frac{1}{3}\left(\left|\sqrt{\alpha_{qp}} - \sqrt{\alpha_p{}^+}\right| + \left|\sqrt{\beta_{qp}} - \sqrt{\beta_p{}^+}\right| + \left|\sqrt{\gamma_{qp}} - \sqrt{\gamma_p{}^+}\right|\right)\right]}{1 - \exp(-1)}, \tag{21}$$

where $d\max_{ap} \in [0, 1]$, $1 \le q \le r$, and $1 \le p \le s$.

Step 8: Based on the normalized crisp weight $\overline{cw}_p$ of attribute φ_p obtained in **Step 4**, where $1 \le p \le s$, and the obtained normalized exponential Hamming distance $d\max_{qp}$ obtained in **Step 7**, where $1 \le q \le r$ and $1 \le p \le s$, compute the weighted score (WTS) ws_q of alternative ω_q, where $ws_q = \sum_{p=1}^{s}\left(d\max_{qp} \times \overline{cw}_p\right)$ [5] and $1 \le q \le r$. The smaller the

WTS ws_q, the better the preference order (PFO) of alternative ω_q, where $1 \le q \le r$.

Example 4.1. Let φ_1, φ_2, φ_3, φ_4 and φ_5 be five attributes, which are all benefit-type attributes. Let ω_1, ω_2, ω_3, ω_4 and ω_5 be five alternatives. Let $\tilde{f}_1$, $\tilde{f}_2$, $\tilde{f}_3$, $\tilde{f}_4$ and $\tilde{f}_5$ be the IVIF weights of the attributes φ_1, φ_2, φ_3, φ_4 and φ_5 given by the DCMK, respectively, where $\tilde{f}_1 = ([0.75,0.86],[0.06,0.13])$, $\tilde{f}_2 = ([0.68,0.82],[0.08,0.16])$, $\tilde{f}_3 = ([0.39,0.57],[0.22,0.38])$, $\tilde{f}_4 = ([0.18,0.34],[0.43,0.62])$ and $\tilde{f}_5 = ([0.42,0.60],[0.18,0.34])$. The DCMX $\tilde{A} = (\tilde{\pi}_{qp})_{5\times5} = \left(\left([\tilde{d}^L_{qp}, \tilde{d}^U_{qp}], [\tilde{g}^L_{qp}, \tilde{g}^U_{qp}]\right)\right)_{5\times5}$ given by the DCMK is shown as follows:

$$\tilde{A} = (\tilde{\pi}_{qp})_{5\times5} = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & \varphi_5 \end{array} \\ \begin{pmatrix} ([0,0.10],[0,0.55]) & ([0.10,0.35],[0.60,0.65]) & ([0,0.15],[0.20,0.50]) & ([0,0.10],[0,0.55]) & ([0,0.10],[0.10,0.30]) \\ ([0,0.15],[0.20,0.50]) & ([0.10,0.15],[0.30,0.65]) & ([0,0.25],[0.40,0.50]) & ([0,0.25],[0.40,0.50]) & ([0,0.30],[0.20,0.60]) \\ ([0.85,0.90],[0.03,0.08]) & ([0.85,0.91],[0.04,0.08]) & ([0.82,0.88],[0.04,0.10]) & ([0.83,0.89],[0.05,0.10]) & ([0.78,0.92],[0.03,0.07]) \\ ([0.58,0.63],[0.26,0.31]) & ([0.49,0.54],[0.36,0.41]) & ([0.44,0.50],[0.38,0.44]) & ([0.25,0.30],[0.59,0.64]) & ([0.34,0.40],[0.47,0.53]) \\ ([0.42,0.60],[0.28,0.35]) & ([0.40,0.50],[0.40,050]) & ([0.38,0.49],[0.40,0.51]) & ([0.20,0.28],[0.70,0.72]) & ([0.20,0.30],[0.50,0.60]) \end{pmatrix} \end{array}$$

Table 1 makes a comparison of the PFOs of the alternatives obtained by various MADM methodologies for ***Example 4.1***. From Table 1, we can observe that Chen and Chen's MADM methodology [3] gets the PFO "$\omega_3 \succ \omega_4 \succ \omega_5 \succ \omega_1 = \omega_2$" of the alternatives ω_1, ω_2, ω_3, ω_4 and ω_5, which has the drawback that it is unable to distinguish the PFO between the alternatives ω_1 and ω_2 because of the improper design of Chen and Chen's score function [3]. Moreover, we can also observe that Wang and Chen's MADM methodology [12] and Wei et al.'s MADM methodology [13] cannot get the PFO of the alternatives, where they are unable to build the linear programming model which satisfies the constraint "$\sum_{p=1}^{5} w_p^* = 1$" of the objective function, where w_p^* denotes the optimal weight of attribute φ_p and $1 \le q \le 5$. Moreover, we also can observe that Kumar and Chen's MADM methodology [9] and our proposed MADM methodology get the same PFO "$\omega_3 \succ \omega_4 \succ \omega_5 \succ \omega_1 \succ \omega_2$" of the alternatives ω_1, ω_2, ω_3, ω_4 and ω_5. Hence, our proposed MADM methodology can overcome the drawbacks of the MADM methodologies presented in [3], [12] and [13] in this example.

Table 1. A comparison of the PFOs of the alternatives obtained using various MADM methodologies for *Example 4.1*.

Methodologies	PFOs
Chen and Chen's MADM methodology [3]	$\omega_3 \succ \omega_4 \succ \omega_5 \succ \omega_1 = \omega_2$
Kumar and Chen's MADM methodology [9]	$\omega_3 \succ \omega_4 \succ \omega_5 \succ \omega_1 \succ \omega_2$
Wang and Chen's MADM methodology [12]	Cannot get the PFO
Wei et al.'s MADM methodology [13]	Cannot get the PFO
Our proposed MADM methodology	$\omega_3 \succ \omega_4 \succ \omega_5 \succ \omega_1 \succ \omega_2$

Example 4.2. Let φ_1, φ_2, φ_3, φ_4 and φ_5 be five attributes, which are all bene-

fit-type attributes, let ω_1, ω_2, ω_3, ω_4 and ω_5 be five alternatives, and let $\tilde{f}_1$, $\tilde{f}_2$, $\tilde{f}_3$, $\tilde{f}_4$ and $\tilde{f}_5$ be the IVIF weights $\tilde{f}_1$, $\tilde{f}_2$, $\tilde{f}_3$, $\tilde{f}_4$ and $\tilde{f}_5$ of the attributes φ_1, φ_2, φ_3, φ_4 and φ_5 given by the DCMK, respectively, where $\tilde{f}_1 = ([0.40,0.46],[0.48,0.54])$, $\tilde{f}_2 = ([0.40,0.44],[0.50,0.56])$, $\tilde{f}_3 = ([0.20,0.30],[0.35,0.40])$, $\tilde{f}_4 = ([0.18,0.20],[0.35,0.45])$ and $\tilde{f}_5 = ([0.15,0.18],[0.40,0.45])$. The DCMX $\tilde{A} = (\tilde{\pi}_{qp})_{5\times5} = \left(\left([\tilde{d}_{qp}^{L}, \tilde{d}_{qp}^{U}], [\tilde{g}_{qp}^{L}, \tilde{g}_{qp}^{U}]\right)\right)_{5\times5}$ given by the DCMK is shown as follows:

$$\tilde{A} = (\tilde{\pi}_{qp})_{5\times5} = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & \varphi_5 \end{array} \\ \begin{pmatrix} ([0.10,1],[0,0]) & ([0.20,1],[0,0]) & ([0.40,1],[0,0]) & ([0.35,1],[0,0]) & ([0.15,1],[0,0]) \\ ([0.30,1],[0,0]) & ([0.50,1],[0,0]) & ([0.60,1],[0,0]) & ([0.45,1],[0,0]) & ([0.25,1],[0,0]) \\ ([0.70,1],[0,0]) & ([0.80,1],[0,0]) & ([0.90,1],[0,0]) & ([0.60,1],[0,0]) & ([0.40,1],[0,0]) \\ ([0.80,1],[0,0]) & ([0.90,1],[0,0]) & ([0.95,1],[0,0]) & ([0.75,1],[0,0]) & ([0.60,1],[0,0]) \\ ([0.90,1],[0,0]) & ([1,1],[0,0]) & ([1,1],[0,0]) & ([0.90,1],[0,0]) & ([0.80,1],[0,0]) \end{pmatrix} \end{array}$$

Table 2 provides a comparison of the PFOs of the alternatives obtained using various MADM methodologies for ***Example 4.2***. From Table 2, we can observe that Kumar and Chen's MADM methodology [9] gets the PFO "$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5$" for the alternatives ω_1, ω_2, ω_3, ω_4 and ω_5, where this occurs because of the improper design of Kumar and Chen's CNs transformation methodology [9] of IVIF values, as shown in Eqs. (3), (4) and (5). Consequently, Kumar and Chen's MADM methodology [9] has the shortcoming of being unable to distinguish the PFO of the alternatives ω_1, ω_2, ω_3, ω_4 and ω_5 in this example. Moreover, we can also observe that Wang and Chen's MADM methodology [12] and Wei et al.'s MADM methodology [13] cannot get the PFO of the alternatives, where they fail to construct the linear programming model which satisfies the constraint "$\sum_{p=1}^{5} w_p^* = 1$" of the objective function, where w_p^* denotes the optimal weight of attribute φ_p and $1 \le p \le 5$. Moreover, we can also observe that Chen and Chen's MADM methodology [3] and our proposed MADM methodology get the same PFO "$\omega_5 \succ \omega_4 \succ \omega_3 \succ \omega_2 \succ \omega_1$" for the alternatives ω_1, ω_2, ω_3, ω_4 and ω_5. Hence, our proposed MADM methodology can overcome the drawbacks of the MADM methodology presented in [9], [12] and [13] in this example.

Table 2. A comparison of the PFOs of the alternatives obtained using various MADM methodologies for *Example 4.2*.

Methodologies	PFOs
Chen and Chen's MADM methodology [3]	$\omega_5 \succ \omega_4 \succ \omega_3 \succ \omega_2 \succ \omega_1$
Kumar and Chen's MADM methodology [9]	$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5$
Wang and Chen's MADM methodology [12]	Cannot get the PFO
Wei et al.'s MADM methodology [13]	Cannot get the PFO
Our proposed MADM methodology	$\omega_5 \succ \omega_4 \succ \omega_3 \succ \omega_2 \succ \omega_1$

5. CONCLUSIONS

In this paper, we have proposed a new MADM methodology based on our proposed CNs transformation methodology for IVIF values, our developed score function [4] for IVIF values, the IVIF positive ideal solution [11], the SPA theory [16], and the normalized exponential Hamming distance [8] between CNs, where the weights of attributes are represented by IVIF values. From ***Example 4.1***, we can observe that our proposed MADM methodology is able to overcome the shortcomings of Chen and Chen's MADM methodology [3], Wang and Chen's MADM methodology [12] and Wei et al.'s MADM methodology [13]. From ***Example 4.2***, we can observe that our proposed MADM methodology is able to overcome the shortcomings of Kumar and Chen's MADM methodology [9], Wang and Chen's MADM methodology [12] and Wei et al.'s MADM methodology [13]. Our proposed MADM methodology provides us with a very useful way to address MADM problems in IVIF environments.

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