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# Multi-attribute decision-making based on generalized intuitionistic fuzzy weighted Hamy mean aggregation operator for green supplier selection in renewable energy sector

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The selection of the most suitable green suppliers is a crucial challenge in the renewable energy industry and is an important multi-attribute decision-making (MADM) problem. In this paper, we propose the Hamy mean (HM) aggregation operator (AO) for aggregating intuitionistic fuzzy numbers (IFNs) and develop the generalized intuitionistic fuzzy Hamy mean (GIFHM) and the generalized intuitionistic fuzzy weighted Hamy mean (GIFWHM) aggregation operators. As an effective aggregation tool, the HM aggregation operator can capture the correlations among multiple attributes during the aggregation process. Based on the proposed GIFWHM aggregation operator, we propose a new MADM algorithm in the environment of intuitionistic fuzzy numbers (IFNs). The effectiveness of the proposed MADM algorithm is demonstrated through a real-world case study of green supplier selection in the renewable energy sector. The applicability and the reliability of the proposed MADM approach are further examined through the case study. Finally, a comparative analysis is conducted to evaluate the advantages of the proposed MADM algorithm, particularly its effectiveness in addressing sustainability-related considerations in green supplier selection.

***Keywords:*** Green supplier; intuitionistic fuzzy set; renewable energy; Hamy mean; decision-making

## 1. INTRODUCTION

Global concern about environmental conservation has increased as a result of climate change and the accelerating effects of global warming. Consequently, investment in renewable energy sources has increased in recent years, driven by advances in technologies for solar, wind, biomass, and waste-to-energy generation. In today's world, the

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renewable energy sector is experiencing growing demand [1]. As a result, the sector faces considerable challenges, including shortages of components for solar panels, wind turbines, and other renewable energy technologies. These challenges highlight the need for effective green supply chain management. One of the key issues in renewable energy supply chain management is green supplier selection [2]. In the context of sustainable development, selecting appropriate green suppliers can contribute to enhancing supply chain sustainability, optimizing resource utilization, and reducing negative environmental impacts. The selection of strategic green suppliers in renewable energy supply chains can be formulated as a multi-attribute decision-making (MADM) problem. However, identifying the most suitable green suppliers remains a complex and challenging task due to the need to consider multiple and often conflicting criteria.

In MADM, decision-makers (DMs) often encounter difficulties in modeling information because of the complexity of the involved parameters. In practical decision-making environments, the inherent uncertainty of information and the limitations of human cognition make it difficult to express assessment information effectively and properly. To address such issues, Zadeh [3] introduced the concept of fuzzy sets (FSs), which represent uncertain information using membership degrees (MDs) ranging from 0 to 1. Some applications of FSs have been presented in [4, 5]. Atanassov [6] laid the foundation for the development of intuitionistic fuzzy sets (IFSs) by incorporating non-membership degrees (NMDs) into FSs. An IFS uses a membership degree ($u$) and a non-membership degree ($v$) to represent intuitionistic fuzzy (IF) information, where the pair $(u, v)$ is referred to as an intuitionistic fuzzy number (IFN). The development of FSs and their extensions, particularly IFSs, has attracted considerable attention in MADM, especially in the development of aggregation operators (AOs) capable of handling complex and uncertain information more effectively. Various decision-making applications [7, 8, 9, 10, 11, 12, 13, 14, 15] have subsequently been developed within these frameworks. To address MADM problems, conventional methods such as TOPSIS [16], VIKOR [17], ELECTRE [18], TODIM [19] are commonly used to rank alternatives. Kumar and Garg [20] developed an IFN-based decision-making method by integrating the TOPSIS method with the set-pair analysis. Dhankhar and Kumar [21] proposed an MADM method based on a possibility degree measure for IFNs. Wan et al. [22] introduced the best-worst method within the IFN framework. Wang and Tsai [23] proposed a fuzzy MADM method for supplier selection in photovoltaic systems. Cheng and Chen [24] developed a best-worst decision-making method under the IFN framework. Jia et al. [25] developed a TODIM method under the IFN framework for selecting green building material suppliers. Tronnebati et al. [26] developed a decision-making method integrating AHP, TOPSIS, and WASPAS for green supplier selection in the Moroccan automotive industry. Anjum et al. [27] developed an IFN-based MADM method for the sustainable management of energy in smart factories.

In addition, aggregation operators (AOs) can serve as effective tools for addressing decision-making problems. Conventional AOs are generally designed to aggregate a set of numerical values into a single value. However, they are typically restricted to real-number inputs. For example, Yager introduced the ordered weighted averaging (OWA) operator [28], the generalized weighted average (GWA) operator [29], and the power average (PA) operator [30]. Bonferroni [31] introduced the Bonferroni mean (BM) operator, while Sykora [32] proposed the Heronian mean operator. These conventional AOs are initially developed primarily for aggregating real numbers. With the continuous devel-

opment of the AOs, an increasing number of AOs have been extended to deal with intuitionistic fuzzy numbers (IFNs). Xu [33] and Xu and Yager [34] extended the arithmetic and the geometric weighted aggregation operators to the IFN environment and proposed several related operators for situations in which the evaluation information is represented by IFNs. A common characteristic of the input arguments of these AOs [33, 34, 35] is that they are assumed to be independent of one another. In contrast, some IFN-based aggregation operators consider the relationships among input arguments. Zhang [36] proposed several power average aggregation operators of IFNs that can reduce the influence of extreme values caused by biased decision-makers (DMs) and account for mutual support among input arguments. Xu and Yager [37] developed the intuitionistic fuzzy Bonferroni mean (IFBM) aggregation operator, which can capture the relationships among the aggregated IFNs. Liu and Chen [38] developed IF Heronian mean aggregation operators based on the Archimedean t-conorms and t-norms. Kumar and Chen [39] developed an improved IF Einstein aggregation operator and presented an MADM approach within the IFN framework. Garg et al. [40] proposed the IF Schweizer-Sklar weighted prioritized averaging (IFSSWPA) aggregation operator and developed an MADM approach based on their proposed operator within the IFN framework. Hussain et al. [41] proposed the IF Aczel-Alsina weighted Heronian mean (IFAAWHM) aggregation operator and the IF Aczel-Alsina geometric Heronian mean (IFAAGHM) aggregation operator and developed an MADM approach based on these operators for the assessment of solar panels. Senapati et al. [42] developed an MADM approach utilizing the IF Aczel-Alsina weighted geometric (IFAAWG) aggregation operator within the IFN framework.

### 1.1 Motivation of the study

In this paper, we identify certain limitations in the existing score function for intuitionistic fuzzy numbers (IFNs) proposed by Chen and Tan [43]. Therefore, there is a need to develop a new score function to improve the ranking of IFNs. Furthermore, we observe that the MADM algorithms proposed by Hussain et al. [41] based on the IFAAWGHM aggregation operator and the IFAAWHM aggregation operator, the MADM algorithm proposed by Senapati et al. [42] based on the IFAAWG aggregation operator, and the MADM algorithm proposed by Garg et al. [40] based on the IFSSWPA aggregation operator may occasionally fail to determine the ranking order (RO) of alternatives. In order to overcome these limitations of the existing MADM algorithms presented in [40, 41, 42], there is a need to develop a new MADM algorithm that can effectively determine the ranking order of alternatives in such situations.

### 1.2 Contributions of the study

In this paper, we propose a new score function for ranking intuitionistic fuzzy numbers (IFNs). Real-world decision-making problems frequently involve interactions among multiple attributes, and the Hamy mean (HM) provides a flexible and general approach for capturing such relationships. Therefore, we extend the HM operator to the IFN framework by employing generalized operational laws (OLs) for IFNs, which can overcome some limitations of conventional operational laws. Accordingly, we propose the generalized intuitionistic fuzzy Hamy mean (GIFHM) and the generalized intuitionistic fuzzy weighted Hamy mean (GIFWHM) aggregation operators for aggregating IFNs. Several

properties and special cases of the proposed operators are also investigated. Moreover, based on the proposed aggregation operators, we develop a new MADM algorithm within the IFN framework. To demonstrate the applicability of the proposed MADM algorithm, we consider a case study involving green supplier selection in the renewable energy sector. Furthermore, we compare the ranking orders (ROs) obtained using the proposed MADM algorithm with those obtained using the MADM algorithms proposed by Hussain et al. [41] based on the IFAAWGHM and IFAAWHM aggregation operators, Senapati et al. [42] based on the IFAAWG aggregation operator, and Garg et al. [40] based on the IF-SSWPA aggregation operator. The proposed MADM algorithm is designed to address the limitations of the existing approaches presented in [40, 41, 42], particularly in cases where these MADM methods fail to determine the ranking orders of alternatives.

### 1.3 Structure of the paper

The remainder of this paper is organized as follows. Section 2, introduces the basic concepts of intuitionistic fuzzy sets (IFSs), including their definitions, properties, comparison methods, and several fundamental concepts related to the Hamy mean (HM) operator. Section 3 presents the proposed score function for intuitionistic fuzzy numbers (IFNs). Section 4 introduces several generalized operational laws (OLs) and develops the generalized intuitionistic fuzzy Hamy mean (GIFHM) operator and the generalized intuitionistic fuzzy weighted Hamy mean (GIFWHM) operator by incorporating the generalized operational laws of IFNs into the HM operator. Section 5 presents a multi-attribute decision-making (MADM) algorithm within the IFN framework based on the proposed operators. Section 6 presents a case study of green supplier selection to demonstrate the applicability of the proposed MADM algorithm. Section 7 provides a comparative analysis to examine the advantages of the proposed MADM algorithm. Finally, Section 8 concludes the paper and summarizes the main findings.

## 2. PRELIMINARIES

**Definition 1.** [6] Let $X$ be a universe of discourse. Then, an intuitionistic fuzzy set $I$ in $X$ is represented by:

$$I = \{\langle \theta_I(x), \vartheta_I(x)\rangle, x \in X\}, \tag{1}$$

where $\theta_I(x)$ and $\vartheta_I(x)$ denote the membership (MD) and non-membership (NMD) functions, respectively, such that $\theta_I(x) \in [0,1]$, $\vartheta_I(x) \in [0,1]$ and $0 \leq \theta_I(x) + \vartheta_I(x) \leq 1, \forall x \in X$. The value $\pi_I(x) = 1 - \theta_I(x) - \vartheta_I(x)$ is called the hesitance degree of $x \in X$. Usually, the pair $\langle \theta, \vartheta \rangle$ is called the IFN.

**Definition 2.** [43] Let $\Upsilon = \langle \theta_\Upsilon, \vartheta_\Upsilon \rangle$ be an IFN. The score function $S(\Upsilon)$ of the IFN $\Upsilon$ is described as below:

$$S(\Upsilon) = \theta_\Upsilon - \vartheta_\Upsilon, \tag{2}$$

where $0 \leq S(\Upsilon) \leq 1$.

**Definition 3.** [44] The Hamy Mean (HM) operator is defined as follows:

$$HM^{(k)}(b_1,b_2,\ldots,b_n)=\frac{\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}}\left(\prod_{j=1}^{k}b_{i_j}\right)^{\frac{1}{k}}}{n_{C_k}}, \tag{3}$$

where $k$ is a parameter $(k=1,2,\ldots,n)$, $(i_1,i_2,\ldots,i_k)$ is k-tuple combinations of $(1,2,\ldots,n)$ and $n_{C_k}$ expresses the binomial coefficient.

The HM operator satisfies the following characteristics:

(i) $\mathrm{HM}^{(k)}(b,b,\ldots,b)=b$;

(ii) If $b_i\le c_i$, then $\mathrm{HM}^{(k)}(b_1,b_2,\ldots,b_n)\le \mathrm{HM}^{(k)}(c_1,c_2,\ldots,c_n)$ for all $i$;

(iii) $\min\limits_i\{b_i\}\le \mathrm{HM}^{(k)}(b_1,b_2,\ldots,b_n)\le \max\limits_i\{b_i\}$

**Definition 4.** [45] A function $\tau:[0,1]\times[0,1]\longrightarrow[0,1]$ is called a t-norm (t-NM) if it satisfies the following conditions:

1. (Boundary) $\tau(0,0)=0$; $\tau(x,1)=\tau(1,x)=x$ for all $x\in[0,1]$
2. (Monotonicity): If $x_1\le y_1$, $x_2\le y_2$ then $\tau(x_1,x_2)\le\tau(y_1,y_2)$.
3. (Commutativity): $\tau(x_1,x_2)=\tau(x_2,x_1)$.
4. (Associativity): $\tau(x_1,\tau(x_2,x_3))=\tau(\tau(x_1,x_2),x_3)$.

Alternatively, the t-conorm (t-CNM) is given by $\delta$ as $\delta(x,y)=1-\tau(1-x,1-y)$.

The strict Archimedian t-CN $\tau$ and t-CNM $\delta$ can be written using the generators $p:[0,1]\to[0,-\infty)$ and $q(x)=p(1-x)$, respectively, as $\tau(x,y)=q^{-1}(q(x)+q(y))$ and $\delta(x,y)=p^{-1}(p(x)+p(y))$.

In the following, we analyze the limitations of the existing score function [43] of IFNs defined in Definition 2.

**Example 1.** Consider two IFNs $\Upsilon_1=\langle 0.3,0.2\rangle$ and $\Upsilon_2=\langle 0.4,0.3\rangle$. To rank IFNs $\Upsilon_1=\langle 0.3,0.2\rangle$ and $\Upsilon_2=\langle 0.4,0.3\rangle$, we compute the score [43] by using Eq. (2) and obtain $S(\Upsilon_1)=\theta_{\Upsilon_1}-\vartheta_{\Upsilon_1}=0.3-0.2=0.1$ and $S(\Upsilon_2)=\theta_{\Upsilon_2}-\vartheta_{\Upsilon_2}=0.4-0.3=0.1$. Since, $S(\Upsilon_1)=S(\Upsilon_2)$, the existing score function $S$ [43] is unable to ranking IFNs $\Upsilon_1=\langle 0.3,0.2\rangle$ and $\Upsilon_2=\langle 0.4,0.3\rangle$.

**Example 2.** Consider two IFNs $\Upsilon_1=\langle 0.78,0.2\rangle$ and $\Upsilon_2=\langle 0.7,0.12\rangle$. To rank IFNs $\Upsilon_1=\langle 0.78,0.2\rangle$ and $\Upsilon_2=\langle 0.7,0.12\rangle$, we compute the score [43] by using Eq. (2) and obtain $S(\Upsilon_1)=\theta_{\Upsilon_1}-\vartheta_{\Upsilon_1}=0.78-0.2=0.58$ and $S(\Upsilon_2)=\theta_{\Upsilon_2}-\vartheta_{\Upsilon_2}=0.7-0.12=0.58$. Since, $S(\Upsilon_1)=S(\Upsilon_2)$, the existing score function $S$ [43] is unable to ranking IFNs $\Upsilon_1=\langle 0.78,0.2\rangle$ and $\Upsilon_2=\langle 0.7,0.12\rangle$.

From the results of Example 1 and Example 2, it is clear that the existing score function $S$ [43] given in Definition 2 has the drawback of occasionally failing to distinguish the ranking of IFNs . Thus, there is a need to develop a new ranking method to overcome the limitations of the existing score function [43].

## 3. THE PROPOSED SCORE FUNCTION OF IFNS

In this section, we introduce a novel score function to overcome the limitations of the existing score function [43] of IFNs.

**Definition 5.** Let $\Upsilon = \langle \theta_\Upsilon, \vartheta_\Upsilon \rangle$ be an IFN. The proposed score function $\mathfrak{S}$ for IFNs is defined as follows:

$$\mathfrak{S}(\Upsilon) = \frac{(1+\theta_\Upsilon - \vartheta_\Upsilon)(1-\pi_\Upsilon)}{2}, \tag{4}$$

where $\pi_\Upsilon = 1 - \theta_\Upsilon - \vartheta_\Upsilon$ and $\mathfrak{S}(\Upsilon) \in [0,1]$.

**Theorem 1.** *Let $\Upsilon_1 = \langle \theta_1, \vartheta_1 \rangle$ and $\Upsilon_2 = \langle \theta_2, \vartheta_2 \rangle$ be two IFNs. Then, $\Upsilon_1 \succ \Upsilon_2$ if $\mathfrak{S}(\Upsilon_1) > \mathfrak{S}(\Upsilon_2)$.*

*Proof.* Straightforward. □

**Property 1.** *Let $\Upsilon = \langle \theta_\Upsilon, \vartheta_\Upsilon \rangle$ be an IFN. Then, the proposed score function $\mathfrak{S}(\Upsilon)$ satisfies the following properties:*

*(i) If $\Upsilon = \langle 0, 1 \rangle$, then $\mathfrak{S}(\Upsilon) = 0$;*

*(ii) If $\Upsilon = \langle 1, 0 \rangle$, then $\mathfrak{S}(\Upsilon) = 1$;*

*(iii) $0 \leq \mathfrak{S}(\Upsilon) \leq 1$.*

**Theorem 2.** *Let $\Upsilon = \langle \theta_\Upsilon, \vartheta_\Upsilon \rangle$ be an IFN. The proposed score function $\mathfrak{S}(\Upsilon)$ increases monotonically as $\theta_\Upsilon$ increases and decreases monotonically as $\vartheta_\Upsilon$ increases.*

*Proof.* On differentiating Eq. (4), w.r.t $\theta_\Upsilon$, we obtain:

$$\frac{\partial \mathfrak{S}}{\partial \theta_\Upsilon} = \frac{2\theta_\Upsilon + 1}{2},$$

Similarly, on differentiating Eq. (4), w.r.t $\vartheta_\Upsilon$, we obtain:

$$\frac{\partial \mathfrak{S}}{\partial \vartheta_\Upsilon} = -\frac{\vartheta_\Upsilon - 1}{2}.$$

Hence, $\mathfrak{S}(\Upsilon)$ increases monotonically as $\theta_\Upsilon$ increases and decreases monotonically as $\vartheta_\Upsilon$ increases. □

**Example 3.** Consider the same IFNs $\Upsilon_1 = \langle 0.3, 0.2 \rangle$ and $\Upsilon_2 = \langle 0.4, 0.3 \rangle$ as given in Example 1. By using Eq. (4), we get

$$\begin{aligned} \mathfrak{S}(\Upsilon_1) &= \frac{(1+\theta_1 - \vartheta_1)(1-\pi_1)}{2} = \frac{(1+0.3-0.2)(1-0.5)}{2} = 0.2750. \\ \mathfrak{S}(\Upsilon_2) &= \frac{(1+\theta_1 - \vartheta_1)(1-\pi_1)}{2} = \frac{(1+0.4-0.3)(1-0.3)}{2} = 0.3850. \end{aligned}$$

Since, $\mathfrak{S}(\Upsilon_1) < \mathfrak{S}(\Upsilon_2)$, we have, $\Upsilon_2 \succ \Upsilon_1$. Hence, the proposed score function overcomes the limitation of the existing score function [43].

**Example 4.** Consider the same IFNs $\Upsilon_1 = \langle 0.78, 0.2\rangle$ and $\Upsilon_2 = \langle 0.7, 0.12\rangle$ as given in Example 2. By using Eq. (4), we get

$$\begin{aligned}\mathfrak{S}(\Upsilon_1) &= \frac{(1+\theta_1-\vartheta_1)(1-\pi_1)}{2} = \frac{(1+0.78-0.2)(1-0.02)}{2} = 0.7742.\\ \mathfrak{S}(\Upsilon_2) &= \frac{(1+\theta_1-\vartheta_1)(1-\pi_1)}{2} = \frac{(1+0.7-0.12)(1-0.18)}{2} = 0.6478.\end{aligned}$$

Since $\mathfrak{S}(\Upsilon_1) > \mathfrak{S}(\Upsilon_2)$, we have $\Upsilon_1 \succ \Upsilon_2$. Hence, the proposed score function $\mathfrak{S}$ can address the limitation of the existing score function [43].

## 4. THE PROPOSED GENERALIZED INTUITIONISTIC FUZZY HAMY MEAN OPERATORS

In this section, some basic generalized operational laws for IFSs are investigated. Moreover, the GIFHM AO and the GIFWHM AO are presented to aggregate a collection of IFNs.

### 4.1 The proposed operational laws of IFNs

**Definition 6.** Let $\Upsilon_1 = \langle \theta_1, \vartheta_1\rangle$, $\Upsilon_2 = \langle \theta_2, \vartheta_2\rangle$ and $\Upsilon = \langle \theta, \vartheta\rangle$ be three IFNs. We have

(i) $\Upsilon_1 \oplus \Upsilon_2 = \left\langle p^{-1}(p(\theta_1)+p(\theta_2)), q^{-1}(q(\vartheta_1)+q(\vartheta_2))\right\rangle$

(ii) $\Upsilon_1 \otimes \Upsilon_2 = \left\langle q^{-1}(q(\theta_1)+q(\theta_2)), p^{-1}(p(\vartheta_1)+p(\vartheta_2))\right\rangle$

(iii) $\lambda\Upsilon = \left\langle p^{-1}(\lambda p(\theta)), q^{-1}(\lambda q(\vartheta))\right\rangle$, $\lambda > 0$

(iv) $\Upsilon^{\lambda} = \left\langle q^{-1}(\lambda q(\theta)), p^{-1}(\lambda p(\vartheta))\right\rangle$, $\lambda > 0$

**Theorem 3.** *If $\Upsilon_1 = \langle \theta_1, \vartheta_1\rangle$, $\Upsilon_2 = \langle \theta_2, \vartheta_2\rangle$ and $\Upsilon = \langle \theta, \vartheta\rangle$ be IFNs. For $\lambda > 0$, $\Upsilon_1 \oplus \Upsilon_2$, $\Upsilon_1 \otimes \Upsilon_2$, $\Upsilon^{\lambda}$ and $\lambda\Upsilon$ are also IFNs.*

*Proof.* By employing Definition 4, we get $0 \le q^{-1}(q(\theta_1)+q(\theta_2)) \le 1$ and $0 \le p^{-1}(p(\vartheta_1)+p(\vartheta_2)) \le 1$. Therefore,

$$\begin{aligned}&q^{-1}\Big(q(\theta_1)+q(\theta_2)\Big) + p^{-1}\Big(p(\vartheta_1)+p(\vartheta_2)\Big)\\ &\le q^{-1}\Big(q(1-\vartheta_1)+q(1-\vartheta_2)\Big) + p^{-1}\Big(p(\vartheta_1)+p(\vartheta_2)\Big)\\ &\le 1 - p^{-1}\Big(p(\vartheta_1)+p(\vartheta_2)\Big) + p^{-1}\Big(p(\vartheta_1)+p(\vartheta_2)\Big)\\ &\le 1.\end{aligned}$$

Therefore, $\Upsilon_1 \otimes \Upsilon_2$ is an IFN. Similarly, we can prove $\Upsilon_1 \oplus \Upsilon_2$, $\Upsilon^{\lambda}$ and $\lambda\Upsilon$ are IFNs. □

### 4.2 Generalized intuitionistic fuzzy Hamy mean operator

In this section, we propose the generalized intuitionistic fuzzy Hamy mean (GIFHM) AO for aggregating IFNs.

**Definition 7.** Let $\Upsilon_1 = \langle \theta_1, \vartheta_1 \rangle$, $\Upsilon_2 = \langle \theta_2, \vartheta_2 \rangle$, ..., $\Upsilon_n = \langle \theta_n, \vartheta_n \rangle$ be $n$ IFNs. The GIFHM AO for aggregating the IFNs $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$ is defined as

$$\mathrm{GIFHM}^{(k)}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) = \frac{\bigoplus\limits_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} \left( \bigotimes\limits_{j=1}^{k} \Upsilon_{i_j} \right)^{\frac{1}{k}}}{n_{C_k}}, \tag{5}$$

where $k$ is a parameter $(k = 1, 2, \ldots, n)$; $(i_1, i_2, \ldots, i_k)$ is k-tuple combinations of $(1, 2, \ldots, n)$ and $n_{C_k}$ expresses the binomial coefficient.

**Theorem 4.** *For a collection of IFNs $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$, the aggregated value from the GIFHM AO is still an IFN and given by:*

$$\begin{aligned} \mathit{GIFHM}^{(k)}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) &= \left\langle p^{-1}\left( \frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left( q^{-1}\left( \frac{1}{k} \sum_{j=1}^{k} q\left(\theta_{i_j}\right) \right) \right) \right), \right. \\ &\quad \left. q^{-1}\left( \frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q\left( p^{-1}\left( \frac{1}{k} \sum_{j=1}^{k} p\left(\vartheta_{i_j}\right) \right) \right) \right) \right\rangle. \end{aligned} \tag{6}$$

*Proof.* By using the operational laws of IFNs, we have

$$\bigotimes_{j=1}^{k} \Upsilon_{i_j} = \left\langle q^{-1}\left( \sum_{j=1}^{k} q\left(\theta_{i_j}\right) \right), p^{-1}\left( \sum_{j=1}^{k} p\left(\vartheta_{i_j}\right) \right) \right\rangle,$$

$$\left( \bigotimes_{j=1}^{k} \Upsilon_{i_j} \right)^{\frac{1}{k}} = \left\langle q^{-1}\left( \frac{1}{k}\left( \sum_{j=1}^{k} q\left(\theta_{i_j}\right) \right) \right), p^{-1}\left( \frac{1}{k}\left( \sum_{j=1}^{k} p\left(\vartheta_{i_j}\right) \right) \right) \right\rangle.$$

Also, we have

$$\begin{aligned} \bigoplus_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} \left( \bigotimes_{j=1}^{k} \Upsilon_{i_j} \right)^{\frac{1}{k}} &= \left\langle p^{-1}\left( \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left( q^{-1}\left( \frac{1}{k}\left( \sum_{j=1}^{k} q\left(\theta_{i_j}\right) \right) \right) \right) \right), \right. \\ &\quad \left. q^{-1}\left( \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q\left( p^{-1}\left( \frac{1}{k}\left( \sum_{j=1}^{k} p\left(\vartheta_{i_j}\right) \right) \right) \right) \right) \right\rangle, \end{aligned}$$

$$\frac{\bigoplus\limits_{\substack{1\le i_1<\ldots\\ <i_k\le n}}\left(\bigotimes\limits_{j=1}^{k}\Upsilon_{i_j}\right)^{\frac{1}{k}}}{n_{C_k}} = \left\langle p^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} p\left(q^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} q\left(\theta_{i_j}\right)\right)\right)\right)\right), q^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} q\left(p^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} p\left(\vartheta_{i_j}\right)\right)\right)\right)\right)\right\rangle.$$

Therefore,

$$\mathrm{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) = \left\langle p^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} p\left(q^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} q\left(\theta_{i_j}\right)\right)\right)\right)\right), q^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} q\left(p^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} p\left(\vartheta_{i_j}\right)\right)\right)\right)\right)\right\rangle.$$

□

In the following, we present some properties of the proposed GIFHM AO given in Eq. (6).

**Theorem 5.** *Let $\Upsilon_i = \langle\theta_i,\vartheta_i\rangle$ and $\Upsilon_i^* = \langle\theta_i^*,\vartheta_i^*\rangle$ $(i = 1,2,\ldots,n)$ be two families of IFNs. We have*

*(P1) (Idempotency:) If $\Upsilon_i = \Upsilon$ for all $i$, then GIFHM$^{(k}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) = \Upsilon$.*

*(P2) (Monotonicity:) If $\theta_i \le \theta_i^*$ and $\vartheta_i \ge \vartheta_i^*$, then GIFHM$^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) \le$ GIFHM$^{(k)}(\Upsilon_1^*,\Upsilon_2^*,\ldots,\Upsilon_n^*)$.*

*(P3) (Boundedness:) Let $\Upsilon^- = \langle\min\limits_i\{\theta_i\},\max\limits_i\{\vartheta_i\}\rangle$ and $\Upsilon^+ = \langle\max\limits_i\{\theta_i\},\min\limits_i\{\vartheta_i\}\rangle$, then*

$$\Upsilon^- \le \mathit{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) \le \Upsilon^+.$$

*(P4) (Commutativity:) For any permutation $(\dot\Upsilon_1,\dot\Upsilon_2,\ldots,\dot\Upsilon_n)$ of $(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n)$, we have*

$$\mathit{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) = \mathit{GIFHM}^{(k)}(\dot\Upsilon_1,\dot\Upsilon_2,\ldots,\dot\Upsilon_n).$$

*Proof.* (P1) Let $\Upsilon = \langle\theta,\vartheta\rangle$ be an IFN, such that $\Upsilon_i = \Upsilon$, for all $i$, then, by using Eq. (6),

we have

$$
\begin{aligned}
& \mathrm{GIFHM}^{(k)}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) \\
= & \left\langle p^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left(q^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} q(\theta)\right)\right)\right)\right),\right. \\
& \left. q^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q\left(p^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} p(\vartheta)\right)\right)\right)\right)\right\rangle \\
= & \left\langle p^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left(q^{-1}\left(\frac{1}{k}(kq(\theta))\right)\right)\right), q^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q\left(p^{-1}\left(\frac{1}{k}(kp(\vartheta))\right)\right)\right)\right\rangle \\
= & \left\langle p^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left(q^{-1}(q(\theta))\right)\right), q^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q\left(p^{-1}(p(\vartheta))\right)\right)\right\rangle \\
= & \left\langle p^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p(\theta)\right), q^{-1}\left(\frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} q(\vartheta)\right)\right\rangle \\
= & \left\langle p^{-1}\left(\frac{1}{n_{C_k}} n_{C_k} p(\theta)\right), q^{-1}\left(\frac{1}{n_{C_k}} n_{C_k} q(\vartheta)\right)\right\rangle \\
= & \left\langle p^{-1}(p(\theta)), q^{-1}(q(\vartheta))\right\rangle \\
= & \langle \theta, \vartheta \rangle = \Upsilon.
\end{aligned}
$$

(P2) Since $\theta_{i_j} \le \theta^*_{i_j}$ and $\vartheta_{i_j} \ge \vartheta^*_{i_j}$, then, for all $j = 1, 2, \ldots, k$, we have $\theta^*_{i_j} \ge \theta_{i_j} \ge 0$ and $\vartheta_{i_j} \ge \vartheta^*_{i_j} \ge 0$. As $p, q$ are increasing and decreasing functions, therefore,

$$
\begin{aligned}
& q(\theta_{i_j}) \ge q(\theta^*_{i_j}) \\
\Rightarrow \quad & \frac{1}{k}\sum_{j=1}^{k} q(\theta_{i_j}) \ge \frac{1}{k}\sum_{j=1}^{k} q(\theta^*_{i_j}) \\
\Rightarrow \quad & q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta_{i_j})\right) \le q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta^*_{i_j})\right) \\
\Rightarrow \quad & p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta_{i_j})\right)\right) \le p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta^*_{i_j})\right)\right) \\
\Rightarrow \quad & \frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta_{i_j})\right)\right) \le \frac{1}{n_{C_k}} \sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta^*_{i_j})\right)\right).
\end{aligned}
$$

Hence

$$p^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta_{i_j})\right)\right)\right) \le p^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} p\left(q^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} q(\theta^*_{i_j})\right)\right)\right). \quad (7)$$

Similarly, we can prove

$$q^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} q\left(p^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} p(\theta_{i_j})\right)\right)\right) \ge q^{-1}\left(\frac{1}{n_{C_k}}\sum_{\substack{1\le i_1<\ldots\\ <i_k\le n}} q\left(p^{-1}\left(\frac{1}{k}\sum_{j=1}^{k} p(\theta^*_{i_j})\right)\right)\right). \quad (8)$$

Let $\Upsilon = \text{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) = \langle\theta,\vartheta\rangle$ and $\Upsilon^* = \text{GIFHM}^{(k)}(\Upsilon^*_1,\Upsilon^*_2,\ldots,\Upsilon^*_n) = \langle\theta^*,\vartheta^*\rangle$, then by using Eqs. (7), (8) and (4), we obtain $\mathfrak{S}(\Upsilon) \le \mathfrak{S}(\Upsilon^*)$.

(P3) Since for all $i$, we have $\Upsilon^- \le \Upsilon_i \le \Upsilon^+$, therefore by using the property (P2), we get $\text{GIFHM}^{(k)}(\Upsilon^-,\Upsilon^-,\ldots,\Upsilon^-) \le \text{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) \le \text{GIFHM}^{(k)}(\Upsilon^+,\Upsilon^+,\ldots,\Upsilon^+)$. Now, based on the property (P1), we get $\Upsilon^- \le \text{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) \le \Upsilon^+$.

(P4) As $(\dot\Upsilon_1,\dot\Upsilon_2,\ldots,\dot\Upsilon_n)$ is a permutation of $(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n)$, therefore,

$$\begin{aligned}
\text{GIFHM}^{(k)}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_n) &= \frac{1}{n_{C_k}}\bigoplus_{\substack{1\le i_1<\ldots\\ <i_k\le n}}\left(\bigotimes_{j=1}^{k}\Upsilon_{i_j}\right)^{\frac{1}{k}}\\
&= \frac{1}{n_{C_k}}\bigoplus_{\substack{1\le i_1<\ldots\\ <i_k\le n}}\left(\bigotimes_{j=1}^{k}\dot\Upsilon_{i_j}\right)^{\frac{1}{k}}\\
&= \text{GIFHM}^{(k,l)}(\dot\Upsilon_{11},\dot\Upsilon_{12},\ldots,\dot\Upsilon_{nm}).
\end{aligned}$$

□

In the following, we present an example to illustrate the applicability of the proposed GIFHM operator.

**Example 5.** Consider a collection of IFNs $\Upsilon_i$ where $i = 1,2,3$, and consider DMs' satisfaction degrees are given as: $\Upsilon_1 = \langle 0.6,0.1\rangle, \Upsilon_2 = \langle 0.5,0.3\rangle$ and $\Upsilon_3 = \langle 0.7,0.2\rangle$. If we utilize the proposed GIFHM AO to the considered data then for $k = 2$ and $q(t) = -\log(t)$, we have

$$\begin{aligned}
\text{GIFHM}^{(2)}(\Upsilon_1,\Upsilon_2,\Upsilon_3) &= \left\langle p^{-1}\left(\frac{1}{3_{C_2}}\sum_{1\le i_1<i_2\le 3} p\left(q^{-1}\left(\frac{1}{2}\left(\sum_{j=1}^{2} q\left(\theta_{i_j}\right)\right)\right)\right)\right),\right.\\
&\quad \left. q^{-1}\left(\frac{1}{3_{C_2}}\sum_{1\le i_1<i_2\le 3} q\left(p^{-1}\left(\frac{1}{2}\left(\sum_{j=1}^{2} p\left(\vartheta_{i_j}\right)\right)\right)\right)\right)\right\rangle\\
&= \langle 0.5979,0.1989\rangle.
\end{aligned}$$

If we take $q(t) = \log\left(\frac{2-t}{t}\right)$ with $k = 2$, then we have

$$\text{GIFHM}^{(2)}(\Upsilon_1, \Upsilon_2, \Upsilon_3) = \langle 0.7835, 0.1972 \rangle$$

### 4.3 Generalized intuitionistic fuzzy weighted Hamy mean operator

In the following, we propose the GIFWHM AO for aggregating IFNs.

**Definition 8.** Let $\Upsilon_1 = \langle \theta_1, \vartheta_1 \rangle$, $\Upsilon_2 = \langle \theta_2, \vartheta_2 \rangle$, ..., $\Upsilon_n = \langle \theta_n, \vartheta_n \rangle$ be $n$ IFNs. The GIFWHM AO for aggregating the IFNs $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$ is defined as

$$\text{GIFWHM}^{(k)}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) = \frac{\bigoplus\limits_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}} \left(1 - \sum_{j=1}^{k} \xi_{i_j}\right) \left(\bigotimes\limits_{j=1}^{k} \Upsilon_{i_j}\right)^{\frac{1}{k}}}{n - 1_{C_k}} \tag{9}$$

where $k$ is a parameter $(k = 1, 2, \ldots, n)$; $(i_1, i_2, \ldots, i_k)$ is k-tuple combinations of $(1, 2, \ldots, n)$ and $n_{C_k}$ expresses the binomial coefficient; $\xi_i$ represents the weight vector corresponding to $\Upsilon_i$ such that $0 \le \xi_i \le 1$ and $\sum_{i=1}^{n} \xi_i = 1$.

**Theorem 6.** *For a collection of IFNs $\Upsilon_i = \langle \theta_i, \vartheta_i \rangle$, the value obtained from GIFWHM AO is still an IFN and given as follows:*

$$\begin{aligned} & \textit{GIFWHM}^{(k)}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) \\ = & \left\langle p^{-1}\left(\frac{1}{n - 1_{C_k}}\left(\sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}}\left((1 - \sum_{j=1}^{k} \xi_{i_j}) p\left(q^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} q\left(\theta_{i_j}\right)\right)\right)\right)\right)\right)\right), \right. \\ & \left. q^{-1}\left(\frac{1}{n - 1_{C_k}}\left(\sum_{\substack{1 \le i_1 < \ldots \\ < i_k \le n}}\left((1 - \sum_{j=1}^{k} \xi_{i_j}) q\left(p^{-1}\left(\frac{1}{k}\left(\sum_{j=1}^{k} p\left(\vartheta_{i_j}\right)\right)\right)\right)\right)\right)\right)\right\rangle. \end{aligned} \tag{10}$$

*Proof.* Proof is similar to the Theorem 4. □

**Example 6.** Consider a collection of IFNs $\Upsilon_i$ where $i = 1, 2, 3$, and consider DMs' satisfaction degrees are given as: $\Upsilon_1 = \langle 0.4, 0.2 \rangle, \Upsilon_2 = \langle 0.5, 0.1 \rangle$ and $\Upsilon_3 = \langle 0.6, 0.2 \rangle$ with weights $\xi_1 = 0.3, \xi_2 = 0.4$ and $\xi_3 = 0.3$, respectively. If we utilize the proposed GIFWHM operator to the considered data then for $k = 2$ and $q(t) = -\log(t)$, we have

$$\begin{aligned} & \text{GIFWHM}^{(2)}(\Upsilon_1, \Upsilon_2, \Upsilon_3) \\ = & \left\langle p^{-1}\left(\frac{1}{3 - 1_{C_2}}\left(\sum_{1 \le i_1 < i_2 \le 3}\left((1 - \sum_{j=1}^{2} \xi_{i_j}) p\left(q^{-1}\left(\frac{1}{2}\left(\sum_{j=1}^{2} q\left(\theta_{i_j}\right)\right)\right)\right)\right)\right)\right), \right. \\ & \left. q^{-1}\left(\frac{1}{3 - 1_{C_2}}\left(\sum_{1 \le i_1 < i_2 \le 3}\left((1 - \sum_{j=1}^{2} \xi_{i_j}) q\left(p^{-1}\left(\frac{1}{2}\left(\sum_{j=1}^{2} p\left(\vartheta_{i_j}\right)\right)\right)\right)\right)\right)\right)\right\rangle. \\ = & \langle 0.4960, 0.1693 \rangle. \end{aligned}$$

## 5. THE PROPOSED MADM ALGORITHM BASED ON THE PROPOSED GIFWHM OPERATOR

In the following, an approach is developed based on the proposed GIFWHM AO and the proposed score function in the environment of IFNs to deal with MADM problems.

Consider a set of $m$ alternatives $I_1, I_2, \ldots, I_m$ which are evaluated on the basis of $n$ attributes $C_1, C_2, \ldots, C_n$ having the weights $\xi_1, \xi_2, \ldots, \xi_n$, respectively, such that $\xi_i > 0$ and $\sum_{i=1}^{n} \xi_i = 1$. Expert evaluates alternative $I_t$ with respect to the attribute $C_i$ by using IFN $\widetilde{\Upsilon}_{ti} = (\langle \widetilde{\theta}_{ti}, \widetilde{\vartheta}_{ti} \rangle)$, where $t = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$, to construct the decision matrix (DMx) $\widetilde{X} = (\widetilde{\Upsilon}_{ti})_{m\times n} = (\langle \widetilde{\theta}_{ti}, \widetilde{\vartheta}_{ti} \rangle)_{m\times n}$, shown as follows:

$$\widetilde{X} = \begin{array}{c} \\ I_1 \\ I_2 \\ \vdots \\ I_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} \widetilde{\Upsilon}_{11} & \widetilde{\Upsilon}_{12} & \ldots & \widetilde{\Upsilon}_{1n} \\ \widetilde{\Upsilon}_{21} & \widetilde{\Upsilon}_{22} & \ldots & \widetilde{\Upsilon}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\Upsilon}_{m1} & \widetilde{\Upsilon}_{m2} & \ldots & \widetilde{\Upsilon}_{mn} \end{pmatrix} \end{array}$$

The procedure of the proposed MADM algorithm is shown as follows:

Step 1: Obtained the normalized DMx (NDMx) $X = (\Upsilon_{ti})_{m\times n} = (\langle \theta_{ti}, \vartheta_{ti} \rangle)_{m\times n}$ by transforming the rating values of the cost (C) type into the benefit (B) type by using the normalization formula:

$$\Upsilon_{ti} = \begin{cases} \langle \theta_{ti}, \vartheta_{ti} \rangle & ; \quad C_i \in B \\ \langle \vartheta_{ti}, \theta_{ti} \rangle & ; \quad C_i \in C \end{cases} \tag{11}$$

Step 2: Based on the the proposed GIFWHM AO given in Eq. (10), compute the average value $\Psi_t$ of each alternative $I_t$, $t = 1, 2, \ldots, m$, where

$$\Psi_t = \langle \theta_t, \vartheta_t \rangle = \text{GIFWHM}(\Upsilon_{t1}, \Upsilon_{t2}, \ldots, \Upsilon_{tn}). \tag{12}$$

Step 3: Based on the obtained average value $\Psi_t$, compute the standard deviation $\sigma_t$ of each alternative $I_t$, $t = 1, 2, \ldots, m$, where

$$\sigma_t = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\mathfrak{S}(\Upsilon_{ti}) - \mathfrak{S}(\Psi_t))^2}, \tag{13}$$

where $\mathfrak{S}(\Upsilon_{ti}) = \frac{(1+\theta_{ti}-\vartheta_{ti})(1-\pi_{ti})}{2}$, $\pi_{ti} = 1 - \theta_{ti} - \vartheta_{ti}$, $\mathfrak{S}(\Psi_t) = \frac{(1+\theta_t-\vartheta_t)(1-\pi_t)}{2}$, $\pi_t = 1 - \theta_t - \vartheta_t$, $t = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$.

Step 4: Based on the obtained average value $\Psi_t$ and the standard deviation $\sigma_t$, compute the positive distance $\xi_{ti}^{+}$ and the negative distance $\xi_{ti}^{-}$ for each IFN $\Upsilon_{ti}$, where

$$\xi_{ti}^{+} = \frac{\max(0, \mathfrak{S}(\Upsilon_{ti}) - \mathfrak{S}(\Psi_t))}{\sigma_t}, \tag{14}$$

$$\xi_{ti}^{-} = \frac{\max(0, \mathfrak{S}(\Psi_t) - \mathfrak{S}(\Upsilon_{ti}))}{\sigma_t}, \tag{15}$$

where $\mathfrak{S}(\Upsilon_{ti}) = \frac{(1+\theta_{ti}-\vartheta_{ti})(1-\pi_{ti})}{2}$, $\pi_{ti} = 1-\theta_{ti}-\vartheta_{ti}$, $\mathfrak{S}(\Psi_t) = \frac{(1+\theta_t-\vartheta_t)(1-\pi_t)}{2}$, $\pi_t = 1-\theta_t-\vartheta_t$, $t = 1,2,\ldots,m$ and $i = 1,2,\ldots,n$.

Step 5: Based on the obtained positive distance $\xi_{ti}^{+}$ and negative distance $\xi_{ti}^{-}$, compute the ranking value $\rho_t$ of each alternative $I_t$, $t = 1,2,\ldots,m$, where

$$\rho_t = \sum_{i=1}^{n} \frac{\xi_{ti}^{+} + 1 - \xi_{ti}^{-}}{2n}. \tag{16}$$

Step 6: Arrange the ranking values $\rho_1$, $\rho_2$, …, $\rho_m$ in a descending order and obtain the ranking order (RO) of the alternatives.

## 6. CASE STUDY OF GREEN SUPPLIER SELECTION IN RENEWABLE ENERGY SECTOR

In the following, the proposed algorithm is utilized to select the best green suppliers for a renewable energy sector that are environmentally conscious by taking into the consideration of the significant factors in economic and environmental aspects.

**Example 7.** As major contributors to climate change and global warming, companies in the energy sector face numerous challenges in measuring, controlling, and reducing the environmental burdens associated with their supply chains. With advances in science and technology, coupled with global trends toward sustainable development, renewable energy sources are increasingly being researched, developed, and utilized. Nowadays, several types of renewable energy are being explored and adopted, including solar, wind, geothermal, and biomass energy. As the renewable energy industry continues to expand, green supplier selection has become a particularly significant issue.

The supplier selection analysis presented in this section applies to a case company involved in renewable energy production. EcoTech Solutions, a pioneering company in the renewable energy sector, is committed to sourcing components and materials from suppliers that prioritize environmental sustainability. Recognizing the importance of green supplier selection in maintaining its commitment to sustainability, EcoTech Solutions has established a Green Procurement Task Force. The task force is responsible for evaluating potential suppliers based on their environmental performance and practices. It consists of procurement specialists, sustainability experts, and engineers with expertise in renewable energy. Following in-depth investigations and evaluations, the task force identified several potential green suppliers for solar panels, wind turbines, and other renewable energy components. Four suppliers, denoted as $I_1, I_2, I_3$, and $I_4$, are shortlisted for further evaluation and selection.

By leveraging the collective knowledge and expertise of its team members, EcoTech Solutions seeks to select suppliers that not only fulfill its quality and performance requirements but also align with its broader mission of promoting environmental sustainability. Through this rigorous supplier selection process, the company aims to promote positive

environmental outcomes and strengthen its position in the renewable energy sector. The following criteria are used to evaluate the alternatives:

**Product Quality** $(C1)$: This criterion encompasses various aspects of product quality, including compliance with established quality standards, quality rejection rates, the supplier's commitment to quality, and the effectiveness of its quality management systems.

**Product Cost** $(C2)$: This criterion represents the total costs incurred when sourcing products from green suppliers. It includes product prices, purchasing costs, procurement costs, and other expenses associated with the acquisition of products.

**Service and Delivery** $(C3)$: This criterion evaluates the quality of services provided by suppliers, their delivery capabilities, their delivery punctuality, and their ability to adapt to changing requirements and circumstances.

**Sustainability** $(C4)$: This criterion evaluates the supplier's commitment to sustainable procurement practices, including minimizing environmental impacts during production, reducing the use of hazardous packaging materials, and increasing the use of recyclable products and packaging.

In this case study, four different green suppliers $I_1, I_2, I_3$ and $I_4$ evaluated by the decision making expert towards the four attributes "Product quality" $(C1)$, "Product cost" $(C2)$, "Service and delivery" $(C3)$ and "Sustainability" $(C4)$ to choose the most efficient green supplier. The weights of the attributes $C_1, C_2, C_3$ and $C_4$ are $\xi_1 = 0.4$, $\xi_2 = 0.1$, $\xi_3 = 0.3$ and $\xi_4 = 0.2$, respectively. The decision making expert evaluates attribute $C_i$ of alternative $I_t$ by utilizing IFN $\widetilde{\Upsilon}_{ti} = (\langle \widetilde{\theta}_{ti}, \widetilde{\vartheta}_{ti} \rangle)$ to build the DMx $\widetilde{X} = (\widetilde{\Upsilon}_{ti})_{4\times 4}$, where

$$\widetilde{X} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle 0.1, 0.3 \rangle & \langle 0.3, 0.2 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0.1, 0.9 \rangle & \langle 0.4, 0.1 \rangle & \langle 0.3, 0.2 \rangle \\ \langle 0.5, 0.2 \rangle & \langle 0.01, 0.1 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0.1, 0.1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.7, 0.3 \rangle & \langle 0.2, 0.1 \rangle \end{pmatrix} \end{array}$$

Step 1: Since the attributes $C_1$, $C_3$ and $C_4$ are the benefit type attributes and the attribute $C_2$ is the cost type attribute, by utilizing Eq. (11), we obtain the NDMx $X = (\Upsilon_{ti})_{4\times 4}$ as follows:

$$X = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle 0.1, 0.3 \rangle & \langle 0.2, 0.3 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0.9, 0.1 \rangle & \langle 0.4, 0.1 \rangle & \langle 0.3, 0.2 \rangle \\ \langle 0.5, 0.2 \rangle & \langle 0.1, 0.01 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0.1, 0.1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.7, 0.3 \rangle & \langle 0.2, 0.1 \rangle \end{pmatrix} \end{array}$$

Step 2: By using the GIFWHM operator defined in Eq. (12) for $q(t) = -\log(t)$, we obtain the average values $\Psi_1$, $\Psi_2$, $\Psi_3$ and $\Psi_4$ of the alternatives $I_1$, $I_2$, $I_3$ and

$I_4$, respectively, where $k = 2$, $\Psi_1 = \langle 0.2831, 0.3\rangle$, $\Psi_2 = \langle 0.4445, 0.1620\rangle$, $\Psi_3 = \langle 0.3805, 0.0871\rangle$, and $\Psi_4 = \langle 0.2946, 0.2019\rangle$.

Step 3: By using Eq. (13), we calculate the standard deviations $\sigma_1$, $\sigma_2$, $\sigma_3$ and $\sigma_4$ of the alternatives $I_1$, $I_2$, $I_3$ and $I_4$, respectively, where $\sigma_1 = 0.1293$, $\sigma_2 = 0.2762$, $\sigma_3 = 0.2723$, and $\sigma_4 = 0.2373$.

Step 4: By using Eqs. (14) and (15), we compute the positive distance $\xi_{ti}^{+}$ and the negative distance $\xi_{ti}^{-}$ for each IFNs $\Upsilon_{ti}$, respectively, where $\xi_{11}^{+} = 0$, $\xi_{12}^{+} = 0$, $\xi_{13}^{+} = 0.0250$, $\xi_{14}^{+} = 0.0127$, $\xi_{21}^{+} = 0$, $\xi_{22}^{+} = 0.1412$, $\xi_{23}^{+} = 0$, $\xi_{24}^{+} = 0$, $\xi_{31}^{+} = 0.0455$, $\xi_{32}^{+} = 0$, $\xi_{33}^{+} = 0.1260$, $\xi_{34}^{+} = 0.0061$, $\xi_{41}^{+} = 0$, $\xi_{42}^{+} = 0.0068$, $\xi_{43}^{+} = 0.1017$, $\xi_{44}^{+} = 0$, $\xi_{11}^{-} = 0.0164$, $\xi_{12}^{-} = 0.0080$, $\xi_{13}^{-} = 0$, $\xi_{14}^{-} = 0$, $\xi_{21}^{-} = 0.0453$, $\xi_{22}^{-} = 0$, $\xi_{23}^{-} = 0.0177$, $\xi_{24}^{-} = 0.0315$, $\xi_{31}^{-} = 0$, $\xi_{32}^{-} = 0.0660$, $\xi_{33}^{-} = 0$, $\xi_{34}^{-} = 0$, $\xi_{41}^{-} = 0.0406$, $\xi_{42}^{-} = 0$, $\xi_{43}^{-} = 0$ and $\xi_{44}^{-} = 0.0252$.

Step 5: By using Eq. (16), we compute the ranking values $\rho_1$, $\rho_2$, $\rho_3$ and $\rho_4$, where $\rho_1 = 0.5017$, $\rho_2 = 0.5058$, $\rho_3 = 0.5135$, and $\rho_4 = 0.5053$.

Step 6: Since $\rho_3 > \rho_2 > \rho_4 > \rho_1$, where $\rho_1 = 0.5017$, $\rho_2 = 0.5058$, $\rho_3 = 0.5135$, and $\rho_4 = 0.5053$, the RO of the alternatives $I_1$, $I_2$, $I_3$ and $I_4$ is "$I_3 \succ I_2 \succ I_4 \succ I_1$". Thus, $I_3$ is the best green supplier among the green suppliers $I_1$, $I_2$, $I_3$ and $I_4$.

### 6.1 Comparison with existing MADM algorithms

In the following, we compare the ROs of the alternatives obtained by the proposed MADM algorithm with the existing MADM algorithms given in [40, 41, 42] for Example 7 to show the practical applicability and the validity of the proposed MADM algorithm.

Table 1 presents a comparison of the ROs of green suppliers $I_1$, $I_2$, $I_3$ and $I_4$ obtained by different MADM algorithms for the Example 7.

**Table 1. A comparative analysis of the ROs of the alternatives derived by various MADM algorithms for Example 7.**

| MADM algorithms | ranking orders |
|---|---|
| Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO | $I_3 \succ I_2 \succ I_4 \succ I_1$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO | $I_3 \succ I_2 \succ I_4 \succ I_1$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO | $I_3 \succ I_2 \succ I_4 \succ I_1$ |
| Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO | $I_3 \succ I_2 \succ I_4 \succ I_1$ |
| The proposed MADM algorithm | $I_3 \succ I_2 \succ I_4 \succ I_1$ |

It is evident from Table 1 that Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO, Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO and the proposed MADM algorithm all produce the same RO "$I_3 \succ I_2 \succ I_4 \succ I_1$" for the green suppliers $I_1$, $I_2$, $I_3$ and $I_4$. Therefore, $I_3$ is the best green supplier among the green suppliers $I_1$, $I_2$, $I_3$ and $I_4$.

## 7. ADVANTAGES OF THE PROPOSED MADM ALGORITHM

In this section , we highlight the advantages of the proposed MADM algorithm compared to the existing MADM algorithms presented in [41, 42] and [40].

**Example 8.** Consider five alternatives $I_1$, $I_2$, $I_3$ and $I_4$ and $I_5$ and four attributes $C_1, C_2, C_3$ and $C_4$ with the weighs $\xi_1 = 0.4$, $\xi_2 = 0.1$, $\xi_3 = 0.3$ and $\xi_4 = 0.2$, respectively. The decision making expert evaluates attribute $C_i$ of alternative $I_t$ by utilizing IFN $\widetilde{\Upsilon}_{ti} = (\langle \widetilde{\theta}_{ti}, \widetilde{\vartheta}_{ti} \rangle)$ to build the DMx $\widetilde{X} = (\widetilde{\Upsilon}_{ti})_{5\times 4}$, where

$$\widetilde{X} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle 0.1, 0.3 \rangle & \langle 0.2, 0.3 \rangle & \langle 0.5, 0.3 \rangle & \langle 0, 1 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0.9, 0.1 \rangle & \langle 0.4, 0.1 \rangle & \langle 0.3, 0.2 \rangle \\ \langle 0.5, 0.2 \rangle & \langle 0.08, 0.01 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0.1, 0.1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.7, 0.3 \rangle & \langle 0.2, 0.1 \rangle \\ \langle 0.5, 0.4 \rangle & \langle 0.1, 0.2 \rangle & \langle 0.3, 0.2 \rangle & \langle 0, 1 \rangle \end{pmatrix} \end{array}$$

To solve this MADM problem, we apply the proposed MADM algorithm, Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO, and Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO. The obtained ROs of the the alternatives $I_1$, $I_2$, $I_3$ and $I_4$ and $I_5$ for Example 8 are shown in Table 2.

**Table 2. A comparison of the alternatives' ROs derived from several MADM algorithms for Example 8.**

| MADM algorithms | ranking orders |
|---|---|
| Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO | $I_3 \succ I_2 \succ I_4 \succ I_5 \succ I_1$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO | $I_3 \succ I_2 \succ I_4 \succ I_5 \succ I_1$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO | $I_3 \succ I_2 \succ I_4 \succ I_5 = I_1$ |
| Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO | $I_3 \succ I_2 \succ I_4 \succ I_5 = I_1$ |
| The proposed MADM algorithm | $I_3 \succ I_2 \succ I_4 \succ I_5 \succ I_1$ |

It is evident from Table 2 that Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO and Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO are unable to differentiate the RO between the alternatives $I_1$ and $I_5$. Moreover, Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO and the proposed MADM algorithm utilizing the GIFWHM AO all produce the same RO "$I_3 \succ I_2 \succ I_4 \succ I_5 \succ I_1$" of the alternatives $I_1$, $I_2$, $I_3$ and $I_4$ and $I_5$. Hence, the proposed MADM algorithm successfully addresses the drawbacks of the Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO and Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO.

**Example 9.** Consider five alternatives $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$ and four attributes $C_1, C_2, C_3$ and $C_4$ with weights $\xi_1 = 0.1$, $\xi_2 = 0.3$, $\xi_3 = 0.4$ and $\xi_4 = 0.2$, respectively. The decision maker evaluates attribute $C_i$ of alternative $I_t$ by utilizing IFN $\widetilde{\Upsilon}_{ti} = (\langle \widetilde{\theta}_{ti}, \widetilde{\vartheta}_{ti} \rangle)$ to build the DMx $\widetilde{X} = (\widetilde{\Upsilon}_{ti})_{5\times 4}$, where

$$\widetilde{X} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle 0.26, 0.4 \rangle & \langle 0.18, 0.8 \rangle & \langle 0.5, 0.4 \rangle & \langle 1, 0 \rangle \\ \langle 0.08, 0.4 \rangle & \langle 0.12, 0.02 \rangle & \langle 0.72, 0.02 \rangle & \langle 0.4, 0.3 \rangle \\ \langle 0.1, 0.3 \rangle & \langle 0.3, 0.02 \rangle & \langle 0.5421, 0 \rangle & \langle 1, 0 \rangle \\ \langle 0.2, 0.2 \rangle & \langle 0.4, 0.4 \rangle & \langle 0.9, 0 \rangle & \langle 0.3, 0.2 \rangle \\ \langle 0.64, 0.3 \rangle & \langle 0.13, 0.3 \rangle & \langle 0.65, 0.3 \rangle & \langle 0.1, 0.4 \rangle \end{pmatrix} \end{array}$$

To solve this MADM problem, we apply the proposed MADM algorithm, Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO, and Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO. The obtained ROs of the the alternatives $I_1$, $I_2$, $I_3$ and $I_4$ and $I_5$ for Example 9 are shown in Table 3.

**Table 3. A comparison of the alternatives' ROs derived from several MADM algorithms for Example 9.**

| MADM algorithms | ranking orders |
|---|---|
| Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO | $I_3 = I_1 \succ I_4 \succ I_5 \succ I_2$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO | $I_3 = I_1 \succ I_4 \succ I_2 \succ I_5$ |
| Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO | $I_3 \succ I_4 \succ I_2 \succ I_5 \succ I_1$ |
| Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO | $I_3 \succ I_4 \succ I_2 \succ I_5 \succ I_1$ |
| The proposed MADM algorithm | $I_3 \succ I_4 \succ I_2 \succ I_5 \succ I_1$ |

It is evident from Table 3 that Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO and Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO are unable to distinguish the RO between the alternatives $I_1$ and $I_3$. Moreover, Senapati et al.'s MADM algorithm [42] utilizing the IFAAWG AO, Hussain et al.'s MADM algorithm [41] utilizing the IFAAWGHM AO and the proposed MADM algorithm utilizing the GIFWHM all produce the same RO "$I_3 \succ I_4 \succ I_2 \succ I_5 \succ I_1$" of the alternatives $I_1$, $I_2$, $I_3$ and $I_4$ and $I_5$. Hence, the proposed MADM algorithm successfully addresses the limitations of the Hussain et al.'s MADM algorithm [41] utilizing the IFAAWHM AO and Garg et al.'s MADM algorithm [40] utilizing the IFSSWPA AO.

## 8. CONCLUSIONS

In this paper, we addressed the objective of incorporating environmental considerations into multi-attribute decision-making (MADM) for green supplier selection in the renewable energy sector. To this end, we extended the Hamy mean (HM) aggregation

operator (AO) to the intuitionistic fuzzy number (IFN) framework and developed the generalized intuitionistic fuzzy Hamy mean (GIFHM) aggregation operator and the generalized intuitionistic fuzzy weighted Hamy mean (GIFWHM) aggregation operator. Several properties and special cases of the proposed operators are also presented to examine their validity and applicability. Based on the proposed aggregation operators, we developed a new MADM algorithm within the IFN framework. The applicability of the proposed MADM algorithm is demonstrated through a case study involving green supplier selection in the renewable energy sector. The case study illustrates the practical applicability of the proposed approach to sustainability-oriented supplier selection and demonstrates its ability to address the associated decision-making challenges. Furthermore, a comparative study was conducted between the proposed MADM algorithm and the existing MADM algorithms, including the algorithm proposed by Garg et al. [40] based on the IFSSWPA aggregation operator, the algorithms proposed by Hussain et al. [41] based on the IFAAWHM aggregation operator and the IFAAWGHM aggregation operator, and the algorithm proposed by Senapati et al. [42] based on the IFAAWG aggregation operator. The comparative results demonstrate the applicability of the proposed MADM algorithm in determining the ranking order (RO) of alternatives. In contrast, the existing algorithms are fail to distinguish the ranking orders of alternatives in certain situations. In future research, the proposed approach can be extended to other types of fuzzy and uncertain environments, such as interval-valued fuzzy sets [46, 47], interval type-2 fuzzy sets [48, 49], Fermatean fuzzy sets [50], spherical fuzzy sets [51], and other generalized fuzzy frameworks.

## REFERENCES

1. A. Aghahosseini, D. Bogdanov, N. Ghorbani, and C. Breyer, "Analysis of 100% renewable energy for iran in 2030: integrating solar pv, wind energy and storage," *International Journal of Environmental Science and Technology*, Vol. 15, 2018, pp. 17–36.

2. B. Masoomi, I. G. Sahebi, M. Fathi, F. Yıldırım, and S. Ghorbani, "Strategic supplier selection for renewable energy supply chain under green capabilities (fuzzy bwm-waspas-copras approach)," *Energy Strategy Reviews*, Vol. 40, 2022, p. 100815.

3. L. A. Zadeh, "Fuzzy sets," *Information and Control*, Vol. 8, no. 3, 1965, pp. 338–353.

4. S. M. Chen and C. M. Huang, "Generating weighted fuzzy rules from relational database systems for estimating null values using genetic algorithms," *IEEE Transactions on Fuzzy Systems*, Vol. 11, no. 4, 2003, pp. 495–506.

5. L. W. Lee and S. M. Chen, "Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations," *Expert Systems with Applications*, Vol. 34, no. 4, 2008, pp. 2763–2771.

6. K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, Vol. 20, no. 1, 1986, pp. 87 – 96.

7. X. Zhang, P. Liu, and Y. Wang, "Multiple attribute group decision making methods based on intuitionistic fuzzy frank power aggregation operators," *Journal of Intelligent & Fuzzy Systems*, Vol. 29, no. 5, 2015, pp. 2235–2246.

8. J. q. Wang, P. Zhou, K. j. Li, H. y. Zhang, and X.-h. Chen, "Multi-criteria decision-making method based on normal intuitionistic fuzzy-induced generalized aggregation operator," *TOP*, Vol. 22, no. 3, 2014, pp. 1103–1122.

9. W. Jiang, B. Wei, X. Liu, X. Li, and H. Zheng, "Intuitionistic fuzzy power aggregation operator based on entropy and its application in decision making," *International Journal of Intelligent Systems*, Vol. 33, no. 1, 2018, pp. 49–67.

10. S. M. Chen and D. C. Chen, "A new multiattribute decision making method based on interval-valued intuitionistic fuzzy values," *Knowledge and Information Systems*, 2025, pp. 1–19.

11. C. Huang, X. Wu, M. Lin, and Z. Xu, "An approach for fuzzy group decision making and consensus measure with hesitant judgments of experts," *Knowledge and Information Systems*, Vol. 66, no. 8, Apr. 2024, pp. 4573–4608.

12. K. Kumar and S. M. Chen, "Multiattribute decision making based on converted decision matrices, probability density functions, and interval-valued intuitionistic fuzzy values," *Information Sciences*, Vol. 554, 2021, pp. 313–324.

13. Rohit, K. Kumar, R. Bhardwaj, and G. Kaur, "Rainfall analysis using FUCOM weighted logarithmic distance measure based on probabilistic dual hesitant preference values," *Water Resources Management*, Vol. 39, 2025, pp. 207–226.

14. H. Garg and R. Arora, "Topsis method based on correlation coefficient for solving decision-making problems with intuitionistic fuzzy soft set information," *AIMS mathematics*, Vol. 5, no. 4, 2020, pp. 2944–2966.

15. V. NAYAGAM *et al.*, "Hybrid fuzzy sets, hybrid intuitionistic fuzzy sets and their applications in mcdm using information imputation." *Journal of Multiple-Valued Logic & Soft Computing*, Vol. 43, no. 3, 2024.

16. C. L. Hwang and K. Yoon, *Methods for Multiple Attribute Decision Making*. Berlin, Heidelberg: Springer Berlin Heidelberg, 1981, Vol. 186, pp. 58–191.

17. S. Opricovic, "Multicriteria optimization of civil engineering systems," *Faculty of Civil Engineering, Belgrade*, Vol. 2, no. 1, 1998, pp. 5–21.

18. R. Benayoun, B. Roy, and B. Sussman, "Electre: Une méthode pour guider le choix en présence de points de vue multiples," *Note de travail*, Vol. 49, 1966, pp. 2–120.

19. L. F. A. M. Gomes *et al.*, "An application of the TODIM method to the multicriteria rental evaluation of residential properties," *European Journal of Operational Research*, Vol. 193, no. 1, 2009, pp. 204–211.

20. K. Kumar and H. Garg, "Connection number of set pair analysis based topsis method on intuitionistic fuzzy sets and their application to decision making," *Applied Intelligence*, Vol. 48, no. 8, Oct 2018, pp. 2112–2119.

21. C. Dhankhar and K. Kumar, "Multi-attribute decision-making based on the advanced possibility degree measure of intuitionistic fuzzy numbers," *Granular Computing*, Vol. 8, 2023, pp. 467–478.

22. S. P. Wan, J. Y. Dong, and S. M. Chen, "A novel intuitionistic fuzzy best-worst method for group decision making with intuitionistic fuzzy preference relations," *Information Sciences*, Vol. 666, 2024, p. 120404.

23. T. C. Wang and S. Y. Tsai, "Solar panel supplier selection for the photovoltaic system design by using fuzzy multi-criteria decision making (mcdm) approaches," *Energies*, Vol. 11, no. 8, 2018, p. 1989.

24. X. Cheng and C. Chen, "Decision making with intuitionistic fuzzy best-worst method," *Expert Systems with Applications*, Vol. 237, 2024, p. 121215.

25. Z. Jia, L. Liu, and Z. Diao, "A group intuitionistic fuzzy exponential TODIM method considering attribute interactions applied to green building material supplier selection," *Sustainability*, Vol. 16, no. 18, 2024, p. 7885.

26. I. Tronnebati, F. Jawab, Y. Frichi, and J. Arif, "Green supplier selection using fuzzy AHP, fuzzy TOPSIS, and fuzzy WASPAS: A case study of the moroccan automotive industry," *Sustainability*, Vol. 16, no. 11, 2024, p. 4580.

27. M. Anjum, N. Kraiem, H. Min, Y. I. Daradkeh, A. K. Dutta, and S. Shahab, "Integrating intuitionistic fuzzy and MCDM methods for sustainable energy management in smart factories," *PloS one*, Vol. 20, no. 1, 2025, p. e0315251.

28. R. R. Yager, "Aggregation operators and fuzzy systems modeling," *Fuzzy sets and systems*, Vol. 67, no. 2, 1994, pp. 129–145.

29. R. R. Yager, "Generalized OWA aggregation operators," *Fuzzy Optimization and Decision Making*, Vol. 3, no. 1, 2004, pp. 93–107.

30. Z. Xu and R. R. Yager, "Power-geometric operators and their use in group decision making," *IEEE Transactions on Fuzzy Systems*, Vol. 18, no. 1, 2009, pp. 94–105.

31. C. Bonferroni, "Sulle medie multiple di potenze," *Bollettino dell'Unione Matematica Italiana*, Vol. 5, no. 3-4, 1950, pp. 267–270.

32. S. Sỳkora, "Mathematical means and averages: Generalized heronian means," *Stan's Library: Castano Primo, Italy*, 2009.

33. Z. Xu, "Intuitionistic fuzzy aggregation operators," *Fuzzy Systems, IEEE Transactions on*, Vol. 15, no. 6, 2007, pp. 1179–1187.

34. Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems*, Vol. 35, no. 4, 2006, pp. 417–433.

35. H. Zhao, Z. Xu, M. Ni, and S. Liu, "Generalized aggregation operators for intuitionistic fuzzy sets," *International Journal of Intelligent Systems*, Vol. 25, no. 1, 2010, pp. 1–30.

36. Z. Zhang, "Generalized Atanassov's intuitionistic fuzzy power geometric operators and their application to multiple attribute group decision making," *Information Fusion*, Vol. 14, no. 4, 2013, pp. 460–486.

37. Z. Xu and R. R. Yager, "Intuitionistic fuzzy Bonferroni means," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, Vol. 41, no. 2, 2011, pp. 568–578.

38. P. D. Liu and S. M. Chen, "Heronian aggregation operators of intuitionistic fuzzy numbers based on the Archimedean t-norm and t-conorm," in *2016 International Conference on Machine Learning and Cybernetics (ICMLC)*, Vol. 2. IEEE, 2016, pp. 686–691.

39. K. Kumar and S. M. Chen, "Multiattribute decision making based on the improved intuitionistic fuzzy Einstein weighted averaging operator of intuitionistic fuzzy values," *Information Sciences*, Vol. 568, 2021, pp. 369–383.

40. H. Garg, Z. Ali, T. Mahmood, M. R. Ali, and A. Alburaikan, "Schweizer-Sklar prioritized aggregation operators for intuitionistic fuzzy information and their application in multi-attribute decision-making," *Alexandria Engineering Journal*, Vol. 67, 2023, pp. 229–240.

41. A. Hussain, K. Ullah, D. Pamucar, I. Haleemzai, and D. Tatić, "Assessment of solar panel using multiattribute decision-making approach based on intuitionistic fuzzy Aczel Alsina Heronian mean operator," *International Journal of Intelligent Systems*, Vol. 2023, no. 1, 2023, p. 6268613.

42. T. Senapati, G. Chen, R. Mesiar, and R. R. Yager, "Intuitionistic fuzzy geometric aggregation operators in the framework of Aczel-Alsina triangular norms and their application to multiple attribute decision making," *Expert Systems with Applications*, Vol. 212, 2023, p. 118832.

43. S. M. Chen and J. M. Tan, "Handling multicriteria fuzzy decision-making problems based on vague set theory," *Fuzzy sets and systems*, Vol. 67, no. 2, 1994, pp. 163–172.

44. T. Hara, M. Uchiyama, and S. E. Takahasi, "A refinement of various mean inequalities," *Journal of Inequalities and Applications*, Vol. 1998, no. 4, 1998, p. 932025.

45. M. Mizumoto, "Pictorial representations of fuzzy connectives, Part I: cases of t-norms, t-conorms and averaging operators," *Fuzzy sets and systems*, Vol. 31, no. 2, 1989, pp. 217–242.

46. S. M. Chen and W. H. Hsiao, "Bidirectional approximate reasoning for rule-based systems using interval-valued fuzzy sets," *Fuzzy Sets and Systems*, Vol. 113, no. 2, 2000, pp. 185–203.

47. I. B. Turksen, "Interval valued fuzzy sets based on normal forms," *Fuzzy sets and systems*, Vol. 20, no. 2, 1986, pp. 191–210.

48. S. M. Chen and L. W. Lee, "Fuzzy multiple criteria hierarchical group decision-making based on interval type-2 fuzzy sets," *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, Vol. 40, no. 5, 2010, pp. 1120–1128.

49. J. M. Mendel, R. I. John, and F. Liu, "Interval type-2 fuzzy logic systems made simple," *IEEE transactions on fuzzy systems*, Vol. 14, no. 6, 2006, pp. 808–821.

50. A. R. Mishra, S. M. Chen, and P. Rani, "Multicriteria decision making based on novel score function of Fermatean fuzzy numbers, the CRITIC method, and the glds method," *Information Sciences*, Vol. 623, 2023, pp. 915–931.

51. P. Rani, S. M. Chen, and A. R. Mishra, "Multi-attribute decision-making based on new distance measure between spherical fuzzy sets, modified symmetry point of criterion weight-determining method, and CRADIS approach" *International Journal of Fuzzy Systems*, Vol. 29, 2027 https://doi.org/10.1007/s40815-025-02157-z.



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